AP Stat Putting It All Together: Probability Models and Sampling Distributions Name ____________________

Part I: Discrete Probability Models

A. Binomial Probability Model **B(n, p)**

Meets the criteria for a binomial distribution but are given a sample size that results in less than 10 successes and less than 10 failures.

Variables

X = the number of _______________ out of________ randomly selected ______________

p = probability of success

q = probability of failure (1 - p)

x = number of trials when the first success occurs

n = sample size

Probability of a success after exactly x outcomes: P(X = x) = $\binom{n}{x} \cdot p^x \cdot q^{n-x}$

Ex 1 A national study shows that approximately 6% of all people have type O-Negative blood. In a study of 16 people, find:

a. The expected number of the 16 people who have O-Negative blood

b. The standard deviation of this sample

c. The probability that none of the 16 have O-negative blood

d. The probability that 1 or two of the 16 have O-Negative blood

Ex 2 A statewide study shows that 22% of all people have been to a minor-league baseball game this year. In a study of 21 people, find:

a. The expected number of the 21 people who have been to a minor league game

b. The standard deviation of this sample

c. The probability that at least 4 of the 21 have been to a minor league game

d. The probability that at most 2 of the 21 have been to a minor league game

B. Approximating the Binomial Distribution using the Normal Model **N(μ, σ)**

Meets the criteria for a binomial distribution but and are given a sample size that results in at least 10 successes and at least 10 failures.

Variables

X = the number of _______________ out of_______ randomly selected ______________

p = probability of success

q = probability of failure (1 - p)

x = number of trials when the first success occurs

n = sample size

Formulas if Binomial ⟹ Normal

Expected value (mean): μ = np

Standard Deviation: $\sigma = \sqrt{npq}$

Using z = $\frac{x-\mu}{\sigma}$ to find a z-score, we can convert this to an area under the normal curve

Ex 1 A national study shows that approximately 6% of all people have type O-Negative blood. In a study of 170 people, find:

a. The expected number of the 170 people who have O-Negative blood

b. The standard deviation of this sample

c. The probability that less than 9 of the 170 have O-negative blood

d. The probability that between 8 and 12 the 170 have O-Negative blood

Ex 2 A statewide study shows that 22% of all people have been to a minor-league baseball game this year. In a study of 250 people, find:

a. The expected number of the 250 who have been to a minor league game

b. The standard deviation of this sample

c. The probability that exactly 60 of the 250 have been to a minor league game

d. The probability that more than 70 of the 250 have been to a minor league game

Part II: Sampling Distributions

A. **Mean Sampling Distribution** N(μ, $\frac{\sigma}{\sqrt{n}}$)

Used when we are given a population mean and standard deviation and wish to base the results of the sample of size n from the results of the population.

The **central limit theorem** states that *regardless* of the shape of the population distribution, the sampling distribution of a mean will be approximately normal. As a result of the central limit theorem:

a. $\mu_{\bar{x}} = \mu$ b. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Variables

μ = Population mean (given in problem)

σ = Population standard deviation

x = Test statistic

n = Sample Size

Formulas

Mean of sample: $\mu_{\bar{x}} = \mu$ Sampling Standard Deviation of the Mean: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Using z = $\frac{x-\mu}{\sigma_{\bar{x}}}$ to find a z-score, we can convert this to an area under the normal curve (probability).

Ex 1 The average number of days that a Ford Escort will run without breaking down is 165 with a standard deviation of 15 days. 64 Ford Escorts were randomly observed for breakdown. Based upon the data find:

a. The mean and standard deviation of the sampling distribution

b. What is the probability that the mean number of days before a breakdown for a randomly selected Ford Escort is between 164.5 and 164.8 days?

c. In the sample of 64 Escorts, what is the probability that the mean number of days before a breakdown is less than 160 days?

Ex 2 The average grade on a state administered Biology exam was shown to be 63.7 with a standard deviation of 11. A random sample of 100 students was taken.

a. What is the mean and standard deviation of the sampling distribution?

b. What is the probability that a randomly selected student will score less than 61 on the Biology exam?

c. What is the probability that the mean grade of the 100 students on the Biology exam was between than 65 and 67.2?

B. Proportion Sampling Distributions

Used when we are given a population proportion and wish to base the results of the sample of size n from the results of the population.

Variables

p = probability of success
q = probability of failure (1 - p)
x = probability to test
n = sample size

Formulas

Mean of the sample proportion: $\mu_{\hat{p}} = p$ Sampling Standard Deviation of the Proportion: $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$

Using z = $\frac{x - \mu_{\hat{p}}}{\sigma_{\hat{p}}}$ to find a z-score, we can convert this to an area under the normal curve

Ex 1

1) A recent survey of Maryland residents shows that 17% believe the Orioles will finish in third place in their division. In a sample of 130 residents find:

a. The mean and standard deviation of the sample proportion

b. The probability that less than 20 of the 130 believe the Orioles will finish in third place

c. The probability that more than 25 of the 130 believe the Orioles will finish in third place

Ex 2

A national survey of Virginia drivers showed that 80% find traffic conditions to be unsatisfactory on I-95. In a sample of 400 drivers find:

a. The mean and standard deviation of the sample proportion.

b. The probability that between than 300 and 315 of the 400 find driving conditions unsatisfactory.

c. The probability that more than 350 of the 400 find driving conditions to be unsatisfactory.