AP STAT Section 8.1: Mean, Variance, and Standard Deviation for Binomial Distributions Name Key

1. In a binomial distribution consisting of 6 trials, how many outcomes have ***exactly one*** success?

SFFFFF FFFSFF $\binom{6}{1} = {}_6C_1 = \frac{6!}{1!5!} = 6$
FSFFFF FFFFSF
FFSFFF FFFFFS

2. In a binomial experiment consisting of twenty trials, how many outcomes have ***exactly ten*** successes?

$\binom{20}{10} = {}_{20}C_{10} = \frac{10!}{10!10!} = 184,756$

3. A carton containing 100 pairs of jeans is inspected. Each item is rated "first quality" or "irregular". After all 100 jeans have been inspected, the number of irregular pair are removed and recorded. Explain why or why not this would be considered a binomial distribution?

1) Fixed number of trials; n = 100 jeans
2) Success = first quality Failure = irregular
3) Condition of jeans independent
4) Prob of success is same from trial to trial. (Don't have to know value!)

Criteria met for Binomial Distribution

4. Suppose that the proportion of people in the country who favor "harsh penalties for habitual criminals" is 0.95. You want to calculate a probability distribution for a random sample of 100 people. Should you use a normal approximation or a binomial distribution? Justify you answer.

1) Fixed number of trials; n = 100 people
2) Success = favors harsh penalties
Failure = does not favor harsh penalties
3) Opinions are independent
4) Prob of success remains the same
$p = .95$ $q = .05$ $B(100, .95)$

$np \geq 10$ $nq \geq 10$
$100(.95) \geq 10$ $100(.05) \geq 10$
$95 \geq 10$ ✓ $5 \geq 10$ ✗

Normal Approximation Not Appropriate

5. Suppose a poll of 20 voters is taken in a large city. The purpose is to determine x, the number who favor a certain candidate for mayor. Suppose that 60% of all the city's voters favor the candidate.

a. Write the ***correct notation*** for this type of distribution. Justify your answer.

1) Fixed number of trials; n = 20 voters
2) Success = favors candidate
Failure = does not favor candidate
3) Opinions are independent
4) Prob of success remains the same
$p = .60$ $q = .40$ $B(20, .60)$

$np \geq 10$ $nq \geq 10$
$20(.6) \geq 10$ $20(.4) \geq 10$
$12 \geq 10$ ✓ $8 \geq 10$ ✗

Normal Approximation Not Appropriate

Use the correct distribution to calculate the following probabilities. Make a probability statement for each and show work and a sketch when necessary. $B(20, .6)$

b. that at most 10 voters favor the candidate

$P(X \leq 10) = .2447$

c. that more than 12 voters favor the candidate

$P(X > 12) = 1 - P(X \leq 11) = .4159$

d. that exactly 11 voters favor the candidate

$P(X = 11) = .1597$

6. One in every four customers entering a grocery store between 5 pm and 7 pm use an express checkout. You randomly select 5 customers and ask them if they used the express checkout.

a. What is the ***correct notation*** to describe this distribution? Justify your answer.

1) Fixed number of trials; $n = 5$ customers
2) Success = uses express checkout
   Failure = does not use express checkout
3) Choices are independent
4) Prob of success remains the same

$p = .25 \quad q = .75 \quad B(5, .25)$

$np \geq 10$

$5(.25) \geq 10$

$1.25 \geq 10$ ✗

Normal Approximation Not Appropriate

b. Use the correct distribution to determine the probability that exactly two customers among the five will use the express checkout.

$P(X = 2) = .2637$

c. Use the correct distribution to determine the probability that at most one customer among the five will use the express checkout.

$P(X \leq 1) = .6328$

d. Use the correct distribution to determine the probability that exactly two customers among the five will not use the express checkout.

$P(X = 3) = .0879$

7. If the temperature in Florida falls below 32° F during certain periods of the year, there is a chance that the citrus crop will be damaged. Suppose that the probability is 0.1 that any given tree will show a measurable damage when the temperature falls to 30°F. A farmer has a grove with 2000 trees.

a. What is the ***correct notation*** to describe this distribution? Justify your answer.

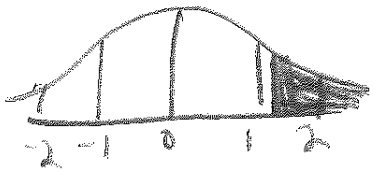
1) Fixed number of trials; $n = 2000$
2) Success = tree damaged
Failure = tree not damaged
3) Damage is independent
4) Prob remains the same
$p = .1 \quad q = .9 \quad B(2000, .1)$

$np \geq 10 \quad nq \geq 10$
$2000(.1) \geq 10 \quad 2000(.9) \geq 10$
$200 \geq 10$ ✓ $\quad 1800 \geq 10$ ✓
Normal Approximation ✓

$\mu = 2000(.1) = 200$
$\sigma = \sqrt{2000(.1)(.9)} = 13.42$
$N(200, 13.42)$

b. Use the correct distribution to determine the probability that at least 215 citrus trees in the farmer's grove will show damage after a night in which the temperature was 30°F. $N(200, 13.42)$

$P(X \geq 215) = P\left(z \geq \frac{215 - 200}{13.42}\right)$
$= P(z \geq 1.12) = .1314$ 13.14%



8. Imprints Galore buys t-shirts (to be imprinted with and item of customer's choice) from a manufacturer with the guarantee that the shirts have been inspected and that no more than 1% are imperfect in any way. The shirts are delivered in boxes of 12. $n = 12 \quad p = .01$

a. Create a probability distribution table for the number of imperfect shirts in a box of 12 shirts. (Use scientific notation if needed for very small probabilities.)

X = number of imperfect shirts in a box of 12 shirts

X	0	1	2	3	4	5	6	7	8	9	10	11	12
P(X)	.8864	.1074	.00597	2×10^{-4}	4.6×10^{-6}	7.4×10^{-8}	9×10^{-10}	8×10^{-12}	5×10^{-14}	2×10^{-16}	6×10^{-19}	1×10^{-21}	1×10^{-24}

*Use $L_1 + L_2$ in 1st Var Stat

b. What is the ***expected number*** of shirts out of a box that will be imperfect? What is the standard deviation?

$\mu_x = 0.12$ *Also meets criteria for binomial dist; $\mu_x = (12)(.01)$ or $\sigma_x = .3447$ or $\sigma_x = \sqrt{(12)(.01)(.99)}$

c. What ***proportion*** of imperfect shirts are found in the distribution between $\mu + \sigma$ and $\mu - \sigma$?

$\mu + \sigma = 0.12 + .3447 = .4647$
$\mu - \sigma = 0.12 - .3447 = -.225$

88.64% of the distribution is between $-.225$ and $.4647$

d. What ***number*** of imperfect shirts are found between $\mu + \sigma$ and $\mu - \sigma$?

0 is the only value for our random variable found between $-.225$ and $.4647$.