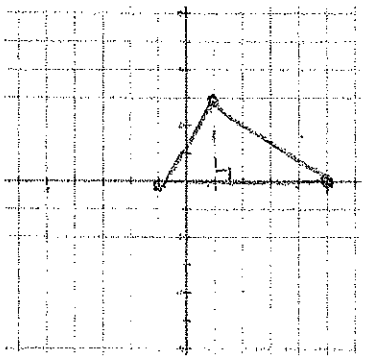
Accel Math II Applying the Determinant of Matrices Name Mine
Unit #1: Matrices

The determinant of a matrix can be used to find the area of a triangle. If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are vertices of a triangle, the area of the triangle is

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

a. Given a triangle with vertices (-1, 0), (1,3), and (5, 0), find the area using the determinant formula. Verify that you are correct using geometric formulas.

Using geometry:



$A = \frac{1}{2}bh = \frac{1}{2}(6)(3) = 9$

Using matrices and technology: $\pm\frac{1}{2}\begin{vmatrix} -1 & 0 & 1 \\ 1 & 3 & 1 \\ 5 & 0 & 1 \end{vmatrix} = -9$ so the area is 9

$-1\begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 5 & 0 \end{vmatrix}$
$-1(3) - 0 + 1(-15) = -18$
$\frac{1}{2}(-18) = -9$

b. Suppose the area of a triangle with vertices (-1, -1), (4, 7), and (9, -6). You find the area of the triangle to be -52.5 and your partner works the same problem and gets +52.5. After checking both solutions, you each have done your work correctly. How can you explain this discrepancy? Points chosen clockwise ⇒ det is neg
Points chosen counterclockwise ⇒ det is pos

c. Suppose another triangle with vertices (1, 1), (4, 2), and (7,3) gives an area of 0. What do you know about the triangle and the points? The points are collinear

d. A gardener is trying to find a triangular area behind his house that encloses 1,750 square feet. He has placed the first two fence posts at (0, 50) and (40, 0). The final fence post is on the property line at y = 100. Find the place where the gardener can place the final fence post.

$1750 = \pm\frac{1}{2}\begin{vmatrix} 0 & 50 & 1 \\ 40 & 0 & 1 \\ x & 100 & 1 \end{vmatrix} = \frac{1}{2}\left[0(0-100) - 50(40-x) + 1(4000-0x)\right]$

$1750 = \pm\frac{1}{2}\left[0 - 2000 + 50x + 4000\right]$

$1750 = \pm 25x + 1000$
$750 = 25x \quad x = 30$

3rd fence placed at (30, 100)

Practice Problems:

Sketch the following triangles. Use **determinants** to find the area. SHOW ALL WORK BELOW.

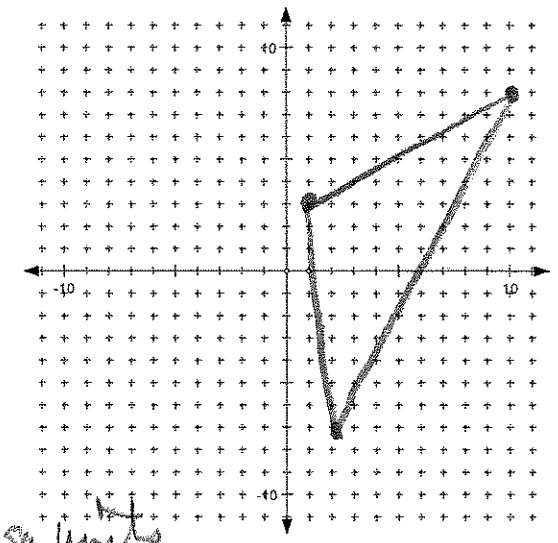
1. (2, -7), (1, 3), (10, 8)

$$A = \pm\frac{1}{2}\begin{vmatrix} 2 & -7 & 1 \\ 1 & 3 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$= \pm\frac{1}{2}\left[2(3-8)-(-7)(1-10)+1(8-30)\right]$$

$$= \pm\frac{1}{2}\left[(2)(-5)+(7)(-9)+(1)(-22)\right]$$

$$= \pm\frac{1}{2}\left[-10-63-22\right] = -\frac{1}{2}\left[-95\right] = 47.5 \text{ sq. units}$$



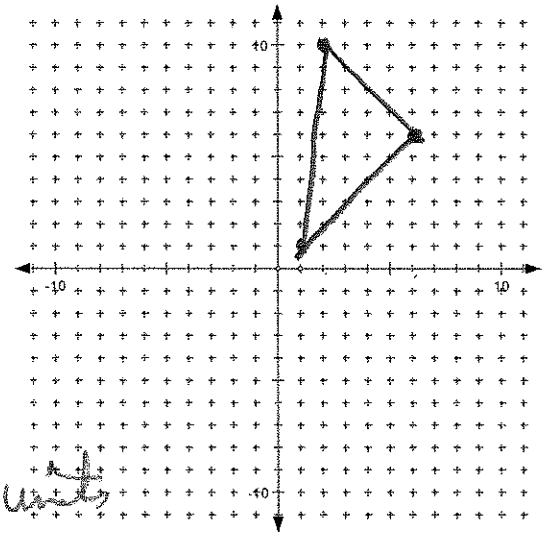
2. (1, 1), (6, 6), and (2, 10)

$$A = \pm\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 6 & 6 & 1 \\ 2 & 10 & 1 \end{vmatrix}$$

$$= \pm\frac{1}{2}\left[1(6-10)-1(6-2)+1(60-12)\right]$$

$$= \pm\frac{1}{2}\left[(1)(-4)-(1)(4)+(1)(48)\right]$$

$$= \pm\frac{1}{2}\left[-4-4+48\right] = \frac{1}{2}\left[40\right] = 20 \text{ sq. units}$$



3. (2,-2), (8, 5), (6,-10)

$$A = \pm\frac{1}{2}\begin{vmatrix} 2 & -2 & 1 \\ 8 & 5 & 1 \\ 6 & -10 & 1 \end{vmatrix}$$

$$= \pm\frac{1}{2}\left[2(5-(-10))-(-2)(8-6)+1(-80-30)\right]$$

$$= \pm\frac{1}{2}\left[2(15)+2(2)+1(-110)\right]$$

$$= \pm\frac{1}{2}\left[30+4-110\right] = -\frac{1}{2}\left[-76\right] = 38 \text{ sq. units}$$