Suggested Methods. Can be proved other ways

① $\sin^2\theta + \sin^2\theta\left(\frac{\cos^2\theta}{\sin^2\theta}\right)$

$\sin^2\theta + \cos^2\theta$

$1 \quad = 1$

② $\sin^2\alpha\left(\frac{1}{\sin^2\alpha}\right) + \sin^2\alpha\left(\frac{1}{\cos^2\alpha}\right)$

$1 + \frac{\sin^2\alpha}{\cos^2\alpha}$

$1 + \tan^2\alpha$

$\sec^2\alpha$

③ $1 - \sin^2\theta - \sin^2\theta$

$1 - 2\sin^2\theta$

④ $\cos^2 t - (1 - \cos^2 t)$

$\cos^2 t - 1 + \cos^2 t$

$2\cos^2 - 1$

⑤ $\cot\beta + \tan\beta$

$\frac{\cos}{\sin} + \frac{\sin}{\cos}$

$\frac{\cos^2 + \sin^2}{\sin\cos}$

$\frac{1}{\sin\cos}$

$\frac{1}{\sin}\ \frac{1}{\cos}$

$\csc\beta\ \sec\beta$

⑥ $\sec^2\alpha + \csc^2\alpha$

$\frac{1}{\cos^2} + \frac{1}{\sin^2}$

$\frac{\sin^2 + \cos^2}{\cos^2\sin^2}$

$\frac{1}{\cos^2\sin^2}$

$\frac{1}{\cos^2}\ \frac{1}{\sin^2}$

$\sec^2\alpha\ \csc^2\alpha$

⑦ $(\cot\beta + \tan\beta)^2$

$\cot^2 + 2\cot\tan + \tan^2$

$\csc^2 - 1 + 2(1) + \sec^2 - 1$

$\csc^2 + \sec^2$

$\frac{1}{\sin^2} + \frac{1}{\cos^2}$

$\frac{\cos^2 + \sin^2}{\sin^2\cos^2}$

$\frac{1}{\sin^2\cos^2}$

$\frac{1}{\sin^2} \cdot \frac{1}{\cos^2} = \csc^2\sec^2$

⑧ * work right side

$\frac{\sec + 1}{\sec - 1}$

$= \frac{\frac{1}{\cos} + 1}{\frac{1}{\cos} - 1}$

$= \frac{\frac{1+\cos}{\cos}}{\frac{1-\cos}{\cos}}$

$\frac{1+\cos}{1-\cos} = \frac{1+\cos}{1-\cos}$