

Suggested Methods. Can be proved other ways

$$\begin{aligned} \textcircled{1} \quad \sin^2 \theta + \sin^2 \theta \left(\frac{\cos^2 \theta}{\sin^2 \theta} \right) \\ \sin^2 \theta + \cos^2 \theta \\ 1 = 1 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \sin^2 \alpha \left(\frac{1}{\sin^2 \alpha} \right) + \sin^2 \alpha \left(\frac{1}{\cos^2 \alpha} \right) \\ 1 + \frac{\sin^2 \alpha}{\cos^2 \alpha} \\ 1 + \tan^2 \alpha \\ \sec^2 \alpha \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad 1 - \sin^2 \theta - \sin^2 \theta \\ 1 - 2\sin^2 \theta \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad \cos^2 t - (1 - \cos^2 t) \\ \cos^2 t - 1 + \cos^2 t \\ 2\cos^2 t - 1 \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad \cot \beta + \tan \beta \\ \frac{\cos}{\sin} + \frac{\sin}{\cos} \\ \frac{\cos^2 + \sin^2}{\sin \cos} \\ \frac{1}{\sin \cos} \\ \frac{1}{\sin} \cdot \frac{1}{\cos} \\ \csc \beta \sec \beta \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad \sec^2 \alpha + \csc^2 \alpha \\ \frac{1}{\cos^2} + \frac{1}{\sin^2} \\ \frac{\sin^2 + \cos^2}{\cos^2 \sin^2} \\ \frac{1}{\cos^2 \sin^2} \\ \frac{1}{\cos^2} \cdot \frac{1}{\sin^2} \\ \sec^2 \alpha \csc^2 \alpha \end{aligned}$$

$$\begin{aligned} \textcircled{7} \quad (\cot \beta + \tan \beta)^2 \\ \cot^2 + 2\cot \tan + \tan^2 \\ \csc^2 - 1 + 2(1) + \sec^2 - 1 \\ \csc^2 + \sec^2 \\ \frac{1}{\sin^2} + \frac{1}{\cos^2} \\ \frac{\cos^2 + \sin^2}{\sin^2 \cos^2} \\ \frac{1}{\sin^2 \cos^2} \\ \frac{1}{\sin^2} \cdot \frac{1}{\cos^2} = \csc^2 \sec^2 \end{aligned}$$

if work right side

$$\begin{aligned} \textcircled{8} \quad \frac{\sec + 1}{\sec - 1} \\ = \frac{\frac{1}{\cos} + 1}{\frac{1}{\cos} - 1} \\ = \frac{1 + \cos}{\cos} \cdot \frac{1 - \cos}{1 - \cos} \\ \frac{1 + \cos}{1 - \cos} = \frac{1 + \cos}{1 - \cos} \end{aligned}$$