Worksheet #1: Verifying Trig Identities

You may choose a different "route". Just make sure your algebra is "sound".

① $\csc\theta \cdot \cos\theta = \cot\theta$

$\frac{1}{\sin\theta} \cdot \frac{\cos\theta}{1}$

$\frac{\cos\theta}{\sin\theta}$

$\cot\theta = \cot\theta$ ✓

② $1 + \tan^2(-\theta) = \sec^2\theta$

* any ratio squared will return a positive value

$1 + \tan^2\theta = \sec^2\theta$

$\sec^2\theta = \sec^2\theta$ ✓

③ $\cos\theta(\tan\theta + \cot\theta) = \csc\theta$

$\frac{\cos\theta}{1} \cdot \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{1} \cdot \frac{\cos\theta}{\sin\theta}$

$\sin\theta + \frac{\cos^2\theta}{\sin\theta}$

$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$

$\frac{1}{\sin\theta}$

$\csc\theta = \csc\theta$ ✓

(4) $\tan\theta \cot\theta - \cos^2\theta = \sin^2\theta$

$\frac{\tan\theta}{1} \cdot \frac{1}{\tan\theta} - \cos^2\theta$

$1 - \cos^2\theta$

$\sin^2\theta = \sin^2\theta$ ✓

(5) $(\sec\theta - 1)(\sec\theta + 1) = \tan^2\theta$

$\sec^2\theta + \sec\theta - \sec\theta - 1$

$\sec^2\theta - 1 = \tan^2\theta$

$\tan^2\theta = \tan^2\theta$ ✓

(6) $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\sec^2\theta - \sec\theta\tan\theta + \sec\theta\tan\theta - \tan^2\theta$

$\sec^2\theta - \tan^2\theta = 1$

$1 = 1$ ✓

(7) $\sin^2\theta(1 + \cot^2\theta) = 1$

$\sin^2\theta(\csc^2\theta) = 1$

$\frac{\sin^2\theta}{1} \cdot \frac{1}{\sin^2\theta} = 1$

$1 = 1$ ✓

(8) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta$

$1 + 1$

$2 = 2$ ✓

(9) $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

$\sec^2\theta(\sec^2\theta - 1)$

$(\sec^2\theta)(\tan^2\theta)$

$(1 + \tan^2\theta)(\tan^2\theta)$

$\tan^2\theta + \tan^4\theta = \tan^4\theta + \tan^2\theta$

(10) $\sec\theta - \tan\theta = \dfrac{\cos\theta}{1+\sin\theta}$

$\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}$

$\dfrac{1-\sin\theta}{\cos\theta}$

$\dfrac{(1+\sin\theta)}{(1+\sin\theta)}\dfrac{(1-\sin\theta)}{\cos\theta}$

$\dfrac{1-\sin^2\theta}{(1+\sin\theta)(\cos\theta)}$

$\dfrac{\cos^2\theta}{(1+\sin\theta)(\cos\theta)} = \dfrac{\cos\theta}{1+\sin\theta}$

(11) $3\sin^2\theta + 4\cos^2\theta = 3 + \cos^2\theta$

$3(1-\cos^2\theta) + 4\cos^2\theta$

$3 - 3\cos^2\theta + 4\cos^2\theta$

$3 + \cos^2\theta = 3 + \cos^2\theta$

(12) $1 - \dfrac{\cos^2\theta}{1+\sin\theta} = \sin\theta$

$1 - \dfrac{1-\sin^2\theta}{1+\sin\theta}$

$1 - \dfrac{(1-\sin\theta)(1+\sin\theta)}{(1+\sin\theta)}$

$1 - (1-\sin\theta)$

$1 - 1 + \sin\theta$

$\sin\theta = \sin\theta$

(13) $\frac{1+\tan\theta}{1-\tan\theta} = \frac{\cot\theta+1}{\cot\theta-1}$ Must work both sides!

$1+\frac{\sin\theta}{\cos\theta}$

$\frac{1-\frac{\sin\theta}{\cos\theta}}{}$

$\frac{\cos\theta+\sin\theta}{\cos\theta}$

$\frac{\cos\theta-\sin\theta}{\cos\theta}$

$\frac{\cos\theta+\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{\cos\theta-\sin\theta}$

$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

$\frac{\frac{\cos\theta}{\sin\theta}+1}{\frac{\cos\theta}{\sin\theta}-1}$

$\frac{\frac{\cos\theta+\sin\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\sin\theta}}$

$\frac{\cos\theta+\sin\theta}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta-\sin\theta}$

$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

(14) $\frac{\sec\theta}{\csc\theta} + \frac{\sin\theta}{\cos\theta} = 2\tan\theta$

$\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}} + \frac{\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1} + \frac{\sin\theta}{\cos\theta}$

$\frac{\sin\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$\tan\theta + \tan\theta$

$2\tan\theta = 2\tan\theta$

(15) $\dfrac{1+\sin\theta}{1-\sin\theta} = \dfrac{\csc\theta+1}{\csc\theta-1}$

$$= \frac{\frac{1}{\sin\theta}+1}{\frac{1}{\sin\theta}-1}$$

$$= \frac{\frac{1+\sin\theta}{\sin\theta}}{\frac{1-\sin\theta}{\sin\theta}}$$

$$\frac{1+\sin\theta}{\sin\theta} \cdot \frac{\sin\theta}{1-\sin\theta}$$

$$\frac{1+\sin\theta}{1-\sin\theta} = \frac{1+\sin\theta}{1-\sin\theta}$$ ✓

(16) $\dfrac{1-\sin\theta}{1+\sin\theta} = (\sec\theta-\tan\theta)^2$ Must work both sides!

$$\frac{(1-\sin\theta)}{(1-\sin\theta)}\frac{(1-\sin\theta)}{(1+\sin\theta)}$$

$$\frac{1-2\sin\theta+\sin^2\theta}{1-\sin^2\theta}$$

$$\frac{1-2\sin^2\theta+\sin^2\theta}{\cos^2\theta}$$

$$(\sec\theta-\tan\theta)(\sec\theta-\tan\theta)$$

$$\sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta$$

$$\frac{1}{\cos^2\theta} - 2\left(\frac{1}{\cos\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right) + \frac{\sin^2\theta}{\cos^2\theta}$$

$$\frac{1}{\cos^2\theta} - \frac{2\sin\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$$

$$\frac{1-2\sin\theta+\sin^2\theta}{\cos^2\theta}$$ ✓

(17) $\tan\theta + \dfrac{\cos\theta}{1+\sin\theta} = \sec\theta$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$$

$$\frac{\sin(1+\sin\theta) + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$\frac{\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$\frac{\sin\theta + 1}{\cos\theta(1+\sin\theta)}$$

$$\frac{1}{\cos\theta} = \sec\theta$$