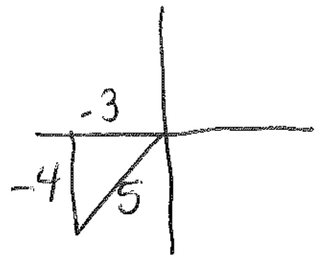
# Practice Worksheet #1: Double + Half Angles

① a) $\sin 2\theta$




$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{-4}{5}\right)\left(\frac{-3}{5}\right) = \boxed{\frac{24}{25}}$$

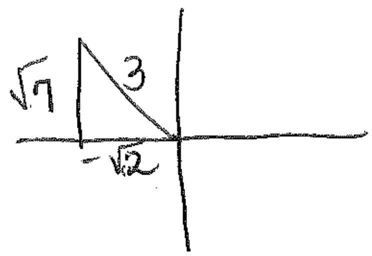
b) $\cos 2\theta$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{-3}{5}\right)^2 - \left(\frac{-4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = \boxed{\frac{-7}{25}}$$

* could use other 2 formulas as well

c) $$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{4}{3}\right)}{\underbrace{1-\left(\frac{4}{3}\right)^2}_{1-\frac{16}{9}}} = \frac{\frac{8}{3}}{\frac{-7}{9}} = \boxed{\frac{-24}{7}}$$

② a) $\sin 2\theta = 2\sin\theta\cos\theta$




$$2\left(\frac{\sqrt{7}}{3}\right)\left(\frac{-\sqrt{2}}{3}\right)$$
$$= \boxed{\frac{-2\sqrt{14}}{9}}$$

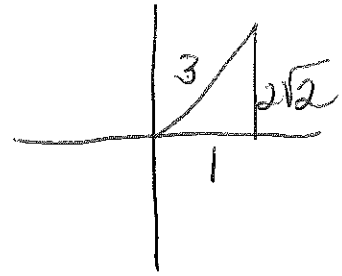
b) $$\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{-\sqrt{2}}{3}\right)^2 - 1 = 2\left(\frac{2}{9}\right) - 1 = \boxed{\frac{-5}{9}}$$

c) $$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{-\sqrt{7}}{\sqrt{2}}\right)}{\underbrace{1-\left(\frac{-\sqrt{7}}{\sqrt{2}}\right)^2}_{1-\frac{7}{2}}} = \frac{\frac{-2\sqrt{7}}{\sqrt{2}}}{\frac{-5}{2}} = \frac{-2\sqrt{7}}{\sqrt{2}} \cdot \frac{-2}{5}$$

$$= \frac{4\sqrt{7}}{5\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{14}}{20}$$
$$= \boxed{\frac{2\sqrt{14}}{5}}$$

③ a) $\sin 2\theta = 2\left(\frac{2\sqrt{2}}{3}\right)\left(\frac{1}{3}\right)$

$= \boxed{\frac{4\sqrt{2}}{9}}$

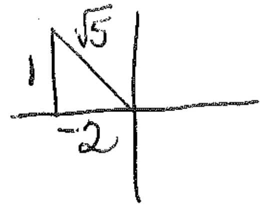



b) $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{1}{3}\right)^2 - \left(\frac{2\sqrt{2}}{3}\right)^2$

$= \frac{1}{9} - \frac{8}{9} = \boxed{\frac{-7}{9}}$

c) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(2\sqrt{2})}{\underbrace{1-(2\sqrt{2})^2}_{1-8}} = \frac{4\sqrt{2}}{-7} = \boxed{-\frac{4\sqrt{2}}{7}}$

~~4~~) no problem




⑤ a) $\sin 2\theta = 2\sin\theta\cos\theta$

$2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{-2}{\sqrt{5}}\right) = \boxed{-\frac{4}{5}}$

b) $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{-2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2$

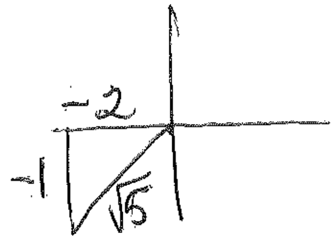
$= \frac{4}{5} - \frac{1}{5} = \boxed{\frac{3}{5}}$

c) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(-\frac{1}{2}\right)}{\underbrace{1-\left(-\frac{1}{2}\right)^2}_{1-\frac{1}{4}}} = \frac{-1}{\frac{3}{4}} = \boxed{-\frac{4}{3}}$

6 a) $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{-1}{\sqrt{5}}\right)\left(\frac{-2}{\sqrt{5}}\right)$

$= \boxed{\frac{4}{5}}$




b) $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{-2}{\sqrt{5}}\right)^2 - \left(\frac{-1}{\sqrt{5}}\right)^2$

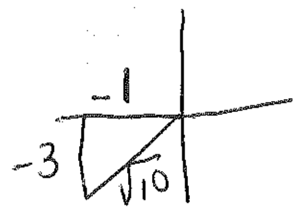
$= \frac{4}{5} - \frac{1}{5} = \boxed{\frac{3}{5}}$

c) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{1}{2}\right)}{\underbrace{1-\left(\frac{1}{2}\right)^2}_{1-\frac{1}{4}}} = \frac{1}{\frac{3}{4}} = \boxed{\frac{4}{3}}$

7 a) $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{-3}{\sqrt{10}}\right)\left(\frac{-1}{\sqrt{10}}\right)$

$= \frac{6}{10} = \boxed{\frac{3}{5}}$



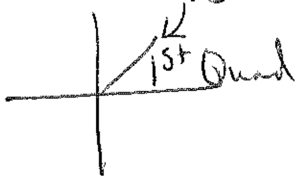

b) $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{-1}{\sqrt{10}}\right)^2 - \left(\frac{-3}{\sqrt{10}}\right)^2 = \frac{1}{10} - \frac{9}{10}$

$= \frac{-8}{10} = \boxed{\frac{-4}{5}}$

c) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(3)}{1-(3)^2} = \frac{6}{-8} = \boxed{\frac{-3}{4}}$

(8) $\sin 22.5° = \sin \frac{45°}{2} = +\sqrt{\frac{1-\cos 45°}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$

$22.5 = \frac{45°}{2}$

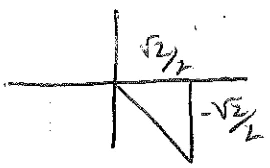



$= \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}}$

$= \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$

(9) $\tan \frac{7\pi}{8} = \tan \frac{\frac{7\pi}{4}}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}} = \frac{-\frac{\sqrt{2}}{2}}{\frac{2+\sqrt{2}}{2}}$

$\frac{7\pi}{8} \cdot 2 = \frac{7\pi}{4}$

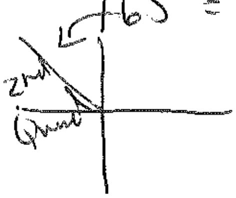



$= \frac{-\sqrt{2}}{\cancel{2}} \cdot \frac{\cancel{2}}{2+\sqrt{2}} = \frac{(-\sqrt{2})}{\underbrace{(2+\sqrt{2}) \cdot (2-\sqrt{2})}_{4-2}} \cdot (2-\sqrt{2})$

$= \frac{-2\sqrt{2}+2}{2} = \boxed{-\sqrt{2}+1 \text{ or } 1-\sqrt{2}}$

(10) $\cos 165° = \cos \frac{330°}{2} = -\sqrt{\frac{1+\cos 330°}{2}} = -\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$

$165° = \frac{330°}{2}$

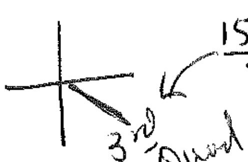



$= -\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{3}}{2} \cdot \frac{1}{2}} = -\sqrt{\frac{2+\sqrt{3}}{4}}$

$= \boxed{\frac{-\sqrt{2+\sqrt{3}}}{2}}$

(11) $\sec \frac{15\pi}{8} \rightarrow \cos \frac{15\pi}{8} = \cos \frac{\frac{15\pi}{4}}{2} = +\sqrt{\frac{1+\cos\frac{15\pi}{4}}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$

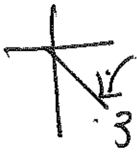
$\frac{15\pi}{8} \cdot 2 = \frac{15\pi}{4}$




$= \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$

$= \frac{\sqrt{2+\sqrt{2}}}{2}$ then flip for sec! $\boxed{\frac{2}{\sqrt{2+\sqrt{2}}}}$

(12) $\sin\frac{-\pi}{8} = \sin\frac{\frac{-\pi}{4}}{2} = -\sqrt{\frac{1-\cos\left(\frac{-\pi}{4}\right)}{2}} = -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$

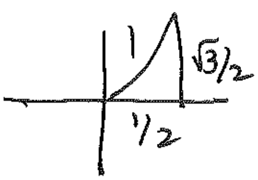


$\frac{-\pi}{8}\cdot 2 = \frac{-\pi}{4}$

3rd Quad

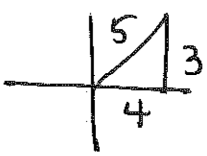
$= -\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2-\sqrt{2}}{4}} = \boxed{-\frac{\sqrt{2-\sqrt{2}}}{2}}$

(13) $\sin\left(2\underbrace{\sin^{-1}\frac{\sqrt{3}}{2}}_{\theta}\right) = \sin(2\theta) = 2\sin\theta\cos\theta$




$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \boxed{\frac{\sqrt{3}}{2}}$

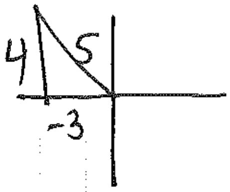
(14) $\cos\left(2\underbrace{\cos^{-1}\frac{4}{5}}_{\theta}\right) = \underbrace{\cos(2\theta)}_{\text{3 formulas to choose from}} = \cos^2\theta - \sin^2\theta$




$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

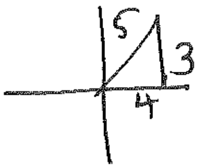
$= \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$

(15) $\tan\left(2\underbrace{\cos^{-1}\left(-\frac{3}{5}\right)}_{\theta}\right) = \tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$




$= \frac{2\left(\frac{-4}{3}\right)}{\underbrace{1-\left(\frac{-4}{3}\right)^2}_{1-\frac{16}{9}}} = \frac{\frac{-8}{3}}{\frac{-7}{9}} = \boxed{\frac{24}{7}}$

(16) $\cos^2\left(\underbrace{\frac{1}{2}\sin^{-1}\frac{3}{5}}_{\theta}\right) = \cos^2\left(\frac{\theta}{2}\right) = \left(+\sqrt{\frac{1+\cos\theta}{2}}\right)^2$




$0° < \theta < 90°$

$0° < \theta/2 < 45°$

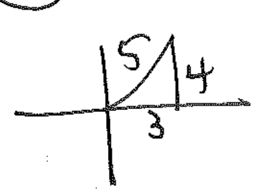
$\downarrow$

1st Quad

$$= \left(+\sqrt{\frac{1+\frac{4}{5}}{2}}\right)^2$$

$$= \frac{\frac{9}{5}}{2} = \boxed{\frac{9}{10}}$$

(17) $\cot^2\left(\underbrace{\frac{1}{2}\tan^{-1}\frac{4}{3}}_{\theta}\right) = \cot^2\left(\frac{\theta}{2}\right)$ work as $\tan^2\left(\frac{\theta}{2}\right)$ then flip!




$$\tan^2\left(\frac{\theta}{2}\right) = \left[\frac{\sin\theta}{1+\cos\theta}\right]^2 = \left[\frac{\frac{4}{5}}{1+\frac{3}{5}}\right]^2$$

$$= \left[\frac{\frac{4}{5}}{\frac{8}{5}}\right]^2 = \left[\frac{4}{5}\cdot\frac{5}{8}\right]^2 = \left[\frac{1}{2}\right]^2 = \frac{1}{4}$$ then flip for cot

$$\boxed{4}$$