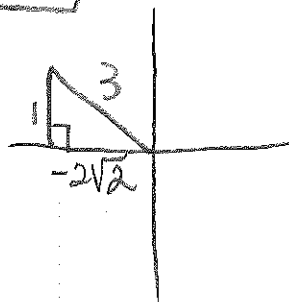


PW # 4: Sum & Difference Formulas

① $\sin \theta = \frac{1}{3}$ θ in Quad II

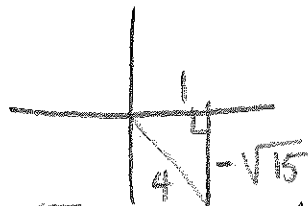


$$\begin{aligned} a) \sin\left(\theta + \frac{\pi}{6}\right) &= \sin\theta \cos\frac{\pi}{6} + \cos\theta \sin\frac{\pi}{6} \\ &= \left(\frac{1}{3}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{-2\sqrt{2}}{3}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{3}}{6} - \frac{2\sqrt{2}}{6} = \boxed{\frac{\sqrt{3} - 2\sqrt{2}}{6}} \end{aligned}$$

$$\begin{aligned} b) \cos\left(\theta - \frac{\pi}{3}\right) &= \cos\theta \cos\frac{\pi}{3} + \sin\theta \sin\frac{\pi}{3} = \left(\frac{-2\sqrt{2}}{3}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= \boxed{\frac{-2\sqrt{2} + \sqrt{3}}{6}} \end{aligned}$$

$$\begin{aligned} c) \tan\left(\theta + \frac{\pi}{4}\right) &= \frac{\tan\theta + \tan\frac{\pi}{4}}{1 - \tan\theta \tan\frac{\pi}{4}} = \frac{-\frac{\sqrt{2}}{4} + 1}{1 - \left(-\frac{\sqrt{2}}{4}\right)(1)} = \frac{-\sqrt{2} + 4}{4 + \sqrt{2}} \\ \frac{1}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} &= \frac{\sqrt{2}}{-4} \\ &= \frac{-\sqrt{2} + 4}{4} \cdot \frac{4}{4 + \sqrt{2}} = \frac{-\sqrt{2} + 4}{4 + \sqrt{2}} \cdot \frac{4 - \sqrt{2}}{4 - \sqrt{2}} = \frac{-4\sqrt{2} + 2 + 16 - 4\sqrt{2}}{16 - 4\sqrt{2} + 4\sqrt{2} - 2} \\ &= \frac{-8\sqrt{2} + 18}{14} = \boxed{\frac{-4\sqrt{2} + 9}{7}} \end{aligned}$$

② $\cos \theta = \frac{1}{4}$ in Quad IV



$$\begin{aligned} a) \sin\left(\theta - \frac{\pi}{6}\right) &= \sin\theta \cos\frac{\pi}{6} - \cos\theta \sin\frac{\pi}{6} = \left(\frac{-\sqrt{15}}{4}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{4}\right)\left(\frac{1}{2}\right) \\ &= \frac{-3\sqrt{5}}{8} - \frac{1}{8} = \boxed{\frac{-3\sqrt{5} - 1}{8}} \end{aligned}$$

$$b) \cos\left(\theta + \frac{\pi}{3}\right) = \cos\theta \cos\frac{\pi}{3} - \sin\theta \sin\frac{\pi}{3} = \left(\frac{1}{4}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{15}}{4}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \boxed{\frac{1 + 3\sqrt{5}}{8}}$$

$$c) \tan\left(\theta - \frac{\pi}{4}\right) = \frac{\tan\theta - \tan\frac{\pi}{4}}{1 + \tan\theta \tan\frac{\pi}{4}} = \frac{-\sqrt{15} - 1}{1 + (-\sqrt{15})(1)} = \frac{-\sqrt{15} - 1}{1 + \sqrt{15}}$$

$$= \frac{-\sqrt{15} - 1}{1 + \sqrt{15}} \cdot \frac{1 + \sqrt{15}}{1 + \sqrt{15}} = \frac{-\sqrt{15} - 15 - 1 - \sqrt{15}}{1 + \sqrt{15} + \sqrt{15} + 15} = \frac{-2\sqrt{15} - 16}{-14} = \boxed{\frac{\sqrt{15} + 8}{7}}$$

$$\textcircled{3} \sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$$

$$\sin\frac{\pi}{2} \cos\theta + \cos\frac{\pi}{2} \sin\theta$$

$$(1)(\cos\theta) + (0)\sin\theta$$

$$\cos\theta = \cos\theta$$

$$\textcircled{4} \cos(\pi - \theta) = -\cos\theta$$

$$\cos\pi \cos\theta + \sin\pi \sin\theta$$

$$(-1)(\cos\theta) + (0)\sin\theta$$

$$-\cos\theta = -\cos\theta$$

$$\textcircled{5} \tan(2\pi - \theta) = -\tan\theta$$

$$\frac{\tan 2\pi - \tan\theta}{1 + \tan 2\pi \tan\theta}$$

$$1 + \tan 2\pi \tan\theta$$

$$0 - \tan\theta$$

$$1 + (0)(\tan\theta)$$

$$\frac{-\tan\theta}{1}$$

$$-\tan\theta = -\tan\theta$$

$$\textcircled{6} \sin(\alpha + \beta) + \sin(\alpha - \beta) = 2\sin\alpha \cos\beta$$

$$\sin\alpha \cos\beta + \cos\alpha \sin\beta + \sin\alpha \cos\beta - \cos\alpha \sin\beta$$

$$2\sin\alpha \cos\beta = 2\sin\alpha \cos\beta$$

$$\textcircled{7} \frac{\sin(\alpha + \beta)}{\sin \alpha \cos \beta} = 1 + \cot \alpha \tan \beta$$

$$\frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \alpha \cos \beta}$$

$$\frac{\sin \alpha \cos \beta}{\sin \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\sin \alpha \cos \beta}$$

$$1 + \cot \alpha \tan \beta = 1 + \cot \alpha \tan \beta$$

$$\textcircled{8} \frac{\cos(\alpha - \beta)}{\sin \alpha \cos \beta} = \cot \alpha + \tan \beta$$

$$\frac{\cos \alpha \cos \beta + \sin \alpha \sin \beta}{\sin \alpha \cos \beta}$$

$$\frac{\cos \alpha \cos \beta}{\sin \alpha \cos \beta} + \frac{\sin \alpha \sin \beta}{\sin \alpha \cos \beta}$$

$$\cot \alpha + \tan \beta = \cot \alpha + \tan \beta$$