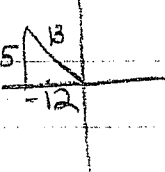
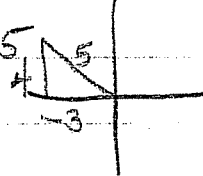


# Practice Worksheet #3

$$\sin \alpha = \frac{5}{13} \quad \cos \alpha = \frac{-12}{13}$$


$$\cos \beta = \frac{3}{5} \quad \sin \beta = \frac{4}{5}$$


$$\begin{aligned} \textcircled{1} \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ &= \left(\frac{5}{13}\right)\left(\frac{3}{5}\right) + \left(\frac{-12}{13}\right)\left(\frac{4}{5}\right) \\ &= \frac{15}{65} + \frac{-48}{65} \\ &= \frac{-33}{65} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ &= \left(\frac{-12}{13}\right)\left(\frac{3}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) \\ &= \frac{-36}{65} + \frac{20}{65} = \frac{-16}{65} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ &= \left(\frac{-12}{13}\right)\left(\frac{3}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) \\ &= \frac{-36}{65} - \frac{20}{65} = \frac{-56}{65} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \sin(\beta - \alpha) &= \sin \beta \cos \alpha - \cos \beta \sin \alpha \\ &= \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) \\ &= \frac{-48}{65} - \frac{15}{65} = \frac{-63}{65} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \\ &= \frac{\frac{-5}{12} + \frac{4}{3}}{1 - \left(\frac{-5}{12}\right)\left(\frac{4}{3}\right)} = \frac{\frac{-5+16}{12}}{1 - \frac{20}{36}} \\ &= \frac{\frac{11}{12}}{\frac{16}{36}} = \frac{11}{12} \cdot \frac{36}{16} = \frac{33}{16} \end{aligned}$$

$$\begin{aligned} \textcircled{6} \csc(\alpha - \beta) &= \frac{1}{\sin \alpha \cos \beta - \cos \alpha \sin \beta} \\ &= \frac{1}{\left(\frac{5}{13}\right)\left(\frac{3}{5}\right) - \left(\frac{-12}{13}\right)\left(\frac{4}{5}\right)} = \frac{1}{\frac{15}{65} + \frac{48}{65}} = \frac{1}{\frac{63}{65}} = \frac{65}{63} \end{aligned}$$

$$\begin{aligned} \textcircled{7} \sec(\beta - \alpha) &= \frac{1}{\cos \beta \cos \alpha + \sin \beta \sin \alpha} = \frac{65}{56} \\ \cos(\beta - \alpha) &= \frac{1}{\left(\frac{3}{5}\right)\left(\frac{-12}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{5}{13}\right)} \\ &= \frac{1}{\frac{-36}{65} + \frac{20}{65}} = \frac{1}{\frac{-16}{65}} \end{aligned}$$

$$\sec \gamma = \frac{65}{56}$$

$$\begin{aligned} \textcircled{8} \cot(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \\ &= \frac{\frac{-5}{12} + \frac{4}{3}}{1 - \left(\frac{-5}{12}\right)\left(\frac{4}{3}\right)} \end{aligned}$$

$$\begin{aligned} &= \frac{\frac{-5+16}{12}}{1 - \frac{20}{36}} = \frac{\frac{11}{12}}{\frac{16}{36}} \\ &= \frac{11}{12} \cdot \frac{36}{16} = \frac{33}{16} \end{aligned}$$

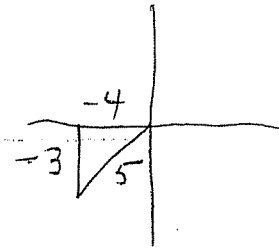
$$\cot \gamma = \frac{16}{33}$$

$$\sin \alpha = \frac{-7}{25}$$

$$\cos \alpha = \frac{-24}{25}$$

$$\sin \beta = \frac{-3}{5}$$

$$\cos \beta = \frac{-4}{5}$$



$$\textcircled{9} \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$= \left(\frac{-24}{25}\right)\left(\frac{-4}{5}\right) - \left(\frac{-7}{25}\right)\left(\frac{-3}{5}\right)$$

$$= \frac{96}{125} - \frac{21}{125} = \frac{75}{125} = \boxed{\frac{3}{5}}$$

$$\textcircled{10} \sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$$

$$= \left(\frac{-7}{25}\right)\left(\frac{-4}{5}\right) + \left(\frac{-3}{5}\right)\left(\frac{-24}{25}\right)$$

$$= \frac{28}{125} + \frac{72}{125} = \frac{100}{125} = \boxed{\frac{4}{5}}$$

$$\textcircled{11} \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = \frac{\frac{7}{24} - \frac{3}{4}}{1 + \left(\frac{7}{24}\right)\left(\frac{3}{4}\right)} = \frac{\frac{11}{24}}{1 + \frac{21}{96}} = \frac{\frac{11}{24}}{\frac{117}{96}} = \frac{11}{24} \cdot \frac{96}{117} = \frac{44}{117}$$

$$\textcircled{12} \cot(\beta - \alpha) = \frac{\tan \beta - \tan \alpha}{1 + \tan \beta \tan \alpha} = \frac{\frac{3}{4} - \frac{7}{24}}{1 + \left(\frac{3}{4}\right)\left(\frac{7}{24}\right)} = \frac{\frac{11}{24}}{1 + \frac{21}{96}} = \frac{\frac{11}{24}}{\frac{117}{96}} = \frac{11}{24} \cdot \frac{96}{117} = \frac{44}{117}$$

$$\frac{1}{\tan(\beta - \alpha)} = \cot \gamma = \frac{1}{\frac{44}{117}} = \boxed{\frac{117}{44}}$$

$$\textcircled{13} \sec(\alpha + \beta) = \frac{1}{\cos(\alpha + \beta)} = \frac{1}{\frac{3}{5}} = \boxed{\frac{5}{3}}$$

$$\textcircled{14} \csc(\alpha - \beta) = \frac{1}{\sin(\alpha - \beta)} = \frac{1}{\frac{-44}{125}} = \boxed{\frac{-125}{44}}$$