

PW #3

$$1) \frac{\sin \alpha \cos \alpha}{1 - \sin^2 \alpha} = \frac{\sin \alpha \cancel{\cos \alpha}}{\cancel{\cos \alpha}} = \frac{\sin \alpha}{\cos \alpha} = \boxed{\tan \alpha}$$

$$2) \frac{1 - \cos^2 \theta}{\sin \theta \cos \theta} = \frac{\sin^2 \theta}{\cancel{\sin \theta} \cos \theta} = \frac{\sin \theta}{\cos \theta} = \boxed{\tan \theta}$$

$$3) \sec \alpha - \sin \alpha \tan \alpha = \frac{1}{\cos \alpha} - \left(\frac{\sin \alpha}{1} \right) \left(\frac{\sin \alpha}{\cos \alpha} \right) \\ = \frac{1}{\cos \alpha} - \frac{\sin^2 \alpha}{\cos \alpha} = \frac{1 - \sin^2 \alpha}{\cos \alpha} = \frac{\cos^2 \alpha}{\cos \alpha} = \boxed{\cos \alpha}$$

$$4) \cos^2 \theta (\cot^2 \theta + 1) = \cos^2 \theta (\csc^2 \theta) = \left(\frac{\cos^2 \theta}{1} \right) \left(\frac{1}{\sin^2 \theta} \right) \\ = \frac{\cos^2 \theta}{\sin^2 \theta} = \boxed{\cot^2 \theta}$$

$$5) \frac{1 - \sin^2 \theta}{1 - \sin \theta} = 1 = \frac{(1 - \cancel{\sin \theta})(1 + \sin \theta)}{(1 - \cancel{\sin \theta})} = 1 = 1 + \sin \theta - 1 \\ = \boxed{\sin \theta}$$

$$6) \frac{\sec^2 \alpha - 1}{\sec \alpha + 1} + 1 = \frac{(\cancel{\sec \alpha + 1})(\sec \alpha - 1)}{(\cancel{\sec \alpha + 1})} + 1 = \sec \alpha - 1 + 1 \\ = \boxed{\sec \alpha}$$

$$7) \frac{\tan \alpha + \cot \alpha}{\sec^2 \alpha} = \frac{\frac{\sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{\sin \alpha}}{\frac{1}{\cos^2 \alpha}} = \frac{\frac{\sin^2 \alpha + \cos^2 \alpha}{\cos \alpha \sin \alpha}}{\frac{1}{\cos^2 \alpha}} \\ = \frac{1}{\cos \alpha \sin \alpha} = \frac{1}{\cos \alpha \sin \alpha} \cdot \frac{\cos^2 \alpha}{1} = \frac{\cos \alpha}{\sin \alpha} = \boxed{\cot \alpha}$$

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$$\begin{aligned} 1) \frac{\sec \theta - \cos \theta}{\tan^2 \theta} &= \frac{\frac{1}{\cos \theta} - \cos \theta}{\frac{\sin^2 \theta}{\cos^2 \theta}} = \frac{1 - \cos^2 \theta}{\frac{\sin^2 \theta}{\cos^2 \theta}} = \frac{\sin^2 \theta}{\frac{\sin^2 \theta}{\cos^2 \theta}} \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta} = \boxed{\cos \theta} \end{aligned}$$

$$\begin{aligned} 2) \cot \theta (\cos \theta \tan \theta + \sin \theta) &= \frac{\cos \theta}{\sin \theta} \left(\frac{\cos \theta}{1} \cdot \frac{\sin \theta}{\cos \theta} + \frac{\sin \theta}{1} \right) \\ &= \frac{\cos \theta}{\sin \theta} \left(\frac{\sin \theta}{1} + \frac{\sin \theta}{1} \right) = \cos \theta + \cos \theta = \boxed{2 \cos \theta} \end{aligned}$$

$$\begin{aligned} 3) \sin \beta + \cos \beta \cot \beta &= \sin \beta + \left(\frac{\cos \beta}{1} \right) \left(\frac{\cos \beta}{\sin \beta} \right) = \sin \beta + \frac{\cos^2 \beta}{\sin \beta} \\ &= \frac{\sin^2 \beta + \cos^2 \beta}{\sin \beta} = \frac{1}{\sin \beta} = \boxed{\csc \beta} \end{aligned}$$

$$\begin{aligned} 4) \frac{\tan^2 \alpha}{\sec \alpha + 1} + 1 &= \frac{\sec^2 \alpha - 1}{\sec \alpha + 1} + 1 = \frac{(\sec \alpha - 1)(\sec \alpha + 1)}{\sec \alpha + 1} + 1 \\ &= \sec \alpha - 1 + 1 = \boxed{\sec \alpha} \end{aligned}$$

$$\begin{aligned} 5) \frac{\sec \beta + \csc \beta}{1 + \tan \beta} &= \frac{\frac{1}{\cos \beta} + \frac{1}{\sin \beta}}{1 + \frac{\sin \beta}{\cos \beta}} = \frac{\frac{\sin \beta + \cos \beta}{\cos \beta \sin \beta}}{\frac{\cos \beta + \sin \beta}{\cos \beta}} \\ &= \frac{\sin \beta + \cos \beta}{\cos \beta \sin \beta} \cdot \frac{\cos \beta}{\cos \beta + \sin \beta} = \frac{1}{\sin \beta} = \boxed{\csc \beta} \end{aligned}$$

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$$(13) \frac{\tan \theta}{1 + \sec \theta} + \frac{1 + \sec \theta}{\tan \theta} = \frac{\frac{\sin \theta}{\cos \theta}}{1 + \frac{1}{\cos \theta}} + \frac{1 + \frac{1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}}$$

$$= \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\cos \theta + 1}{\cos \theta}} + \frac{\frac{\cos \theta + 1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}} = \frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{\cos \theta + 1} + \frac{\cos \theta + 1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta}$$

$$= \frac{(\sin \theta) \cancel{(\cos \theta)}}{(\cos \theta + 1)} + \frac{(\cos \theta + 1) \cancel{(\cos \theta)}}{(\sin \theta) (\cos \theta + 1)} = \frac{\sin^2 \theta + \cos^2 \theta + 2\cos \theta + 1}{(\cos \theta + 1) (\sin \theta)} = \frac{2 + 2\cos \theta}{(\cos \theta + 1) (\sin \theta)}$$

$$= \frac{2(1 + \cancel{\cos \theta})}{(\cos \theta + 1) (\sin \theta)} = \frac{2}{\sin \theta} = \boxed{2 \csc \theta} \text{ when!!}$$

$$(14) \frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{\sin^2 \theta + 1 + 2\cos \theta + \cos^2 \theta}{(1 + \cos \theta) (\sin \theta)} \quad * \text{see \#13}$$

$$= \frac{2 + 2\cos \theta}{(1 + \cos \theta) (\sin \theta)} = \frac{2(1 + \cancel{\cos \theta})}{(1 + \cancel{\cos \theta}) (\sin \theta)} = \frac{2}{\sin \theta} = \boxed{2 \csc \theta}$$

$$(15) \frac{(\cos \theta + \sin \theta)^2 + (\cos \theta - \sin \theta)^2}{(\cos \theta + \sin \theta) (\cos \theta + \sin \theta) (\cos \theta - \sin \theta) (\cos \theta - \sin \theta)} = \frac{\cos^2 \theta + 2\cos \theta \sin \theta + \sin^2 \theta + \cos^2 \theta - 2\cos \theta \sin \theta + \sin^2 \theta + \cos^2 \theta}{2\cos \theta \sin \theta + \sin^2 \theta}$$

$$= 1 + 1 = \boxed{2}$$

$$(16) \underset{\text{FOIL}}{(1 + \tan \beta)^2} + \underset{\text{FOIL}}{(1 - \tan \beta)^2} = 1 + 2\tan \beta + \tan^2 \beta + 1 - 2\tan \beta + \tan^2 \beta$$

$$= 1 + \tan^2 \beta + 1 + \tan^2 \beta = \sec^2 \beta + \sec^2 \beta$$

$$= \boxed{2 \sec^2 \beta}$$