HW #3

1) $\frac{\sin\alpha\cos\alpha}{1-\sin^2\alpha} = \frac{\sin\alpha\cos\alpha}{\cos^2\alpha} = \frac{\sin\alpha}{\cos\alpha} = \boxed{\tan\alpha}$

2) $\frac{1-\cos^2\theta}{\sin\theta\cos\theta} = \frac{\sin^2\theta}{\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} = \boxed{\tan\theta}$

3) $\sec\alpha - \sin\alpha\tan\alpha = \frac{1}{\cos\alpha} - \left(\frac{\sin\alpha}{1}\right)\left(\frac{\sin\alpha}{\cos\alpha}\right)$

$= \frac{1}{\cos\alpha} - \frac{\sin^2\alpha}{\cos\alpha} = \frac{1-\sin^2\alpha}{\cos\alpha} = \frac{\cos^2\alpha}{\cos\alpha} = \boxed{\cos\alpha}$

4) $\cos^2\theta(\cot^2\theta+1) = \cos^2\theta(\csc^2\theta) = \left(\frac{\cos^2\theta}{1}\right)\left(\frac{1}{\sin^2\theta}\right)$

$= \frac{\cos^2\theta}{\sin^2\theta} = \boxed{\cot^2\theta}$

5) $\frac{1-\sin^2\theta}{1-\sin\theta} - 1 = \frac{(1-\sin\theta)(1+\sin\theta)}{(1-\sin\theta)} - 1 = 1+\sin\theta - 1$

$= \boxed{\sin\theta}$

6) $\frac{\sec^2\alpha - 1}{\sec\alpha+1} + 1 = \frac{(\sec\alpha+1)(\sec\alpha-1)}{(\sec\alpha+1)} + 1 = \sec\alpha - 1 + 1$

$= \boxed{\sec\alpha}$

7) $\frac{\tan\alpha+\cot\alpha}{\sec^2\alpha} = \frac{\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}}{\frac{1}{\cos^2\alpha}} = \frac{\frac{\sin^2\alpha+\cos^2\alpha}{\cos\alpha\sin\alpha}}{\frac{1}{\cos^2\alpha}}$

$= \frac{\frac{1}{\cos\alpha\sin\alpha}}{\frac{1}{\cos^2\alpha}} = \frac{1}{\cos\alpha\sin\alpha} \cdot \frac{\cos^2\alpha}{1} = \frac{\cos\alpha}{\sin\alpha} = \boxed{\cot\alpha}$

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8) $\frac{\sec\theta - \cos\theta}{\tan^2\theta} = \frac{\frac{1}{\cos\theta} - \cos\theta}{\frac{\sin^2\theta}{\cos^2\theta}} = \frac{\frac{1-\cos^2\theta}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} = \frac{\frac{\sin^2\theta}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}$

$= \frac{\sin^2\theta}{\cos\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta} = \boxed{\cos\theta}$

9) $\cot\theta(\cos\theta\tan\theta + \sin\theta) = \frac{\cos\theta}{\sin\theta}\left(\frac{\cos\theta}{1}\cdot\frac{\sin\theta}{\cos\theta} + \frac{\sin\theta}{1}\right)$

$= \frac{\cos\theta}{\sin\theta}\left(\frac{\sin\theta}{1} + \frac{\sin\theta}{1}\right) = \cos\theta + \cos\theta = \boxed{2\cos\theta}$

10) $\sin\beta + \cos\beta\cot\beta = \sin\beta + \left(\frac{\cos\beta}{1}\right)\left(\frac{\cos\beta}{\sin\beta}\right) = \sin\beta + \frac{\cos^2\beta}{\sin\beta}$

$= \frac{\sin^2\beta + \cos^2\beta}{\sin\beta} = \frac{1}{\sin\beta} = \boxed{\csc\beta}$

11) $\frac{\tan^2\alpha}{\sec\alpha + 1} + 1 = \frac{\sec^2\alpha - 1}{\sec\alpha + 1} + 1 = \frac{(\sec\alpha - 1)(\sec\alpha + 1)}{\sec\alpha + 1} + 1$

$= \sec\alpha - 1 + 1 = \boxed{\sec\alpha}$

12) $\frac{\sec\beta + \csc\beta}{1 + \tan\beta} = \frac{\frac{1}{\cos\beta} + \frac{1}{\sin\beta}}{1 + \frac{\sin\beta}{\cos\beta}} = \frac{\frac{\sin\beta + \cos\beta}{\cos\beta\sin\beta}}{\frac{\cos\beta + \sin\beta}{\cos\beta}}$

$= \frac{\sin\beta + \cos\beta}{\cos\beta\sin\beta} \cdot \frac{\cos\beta}{\cos\beta + \sin\beta} = \frac{1}{\sin\beta} = \boxed{\csc\beta}$

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(13) $\dfrac{\tan\theta}{1+\sec\theta} + \dfrac{1+\sec\theta}{\tan\theta} = \dfrac{\frac{\sin\theta}{\cos\theta}}{1+\frac{1}{\cos\theta}} + \dfrac{1+\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$

$= \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta+1}{\cos\theta}} + \dfrac{\frac{\cos\theta+1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{\cos\theta}{\cos\theta+1} + \dfrac{\cos\theta+1}{\cos\theta}\cdot\dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{(\sin\theta)(\sin\theta)}{(\sin\theta)(\cos\theta+1)} + \dfrac{(\cos\theta+1)(\cos\theta+1)}{(\sin\theta)(\cos\theta+1)} = \dfrac{\sin^2\theta+\cos^2\theta+2\cos\theta+1}{(\cos\theta+1)(\sin\theta)} = \dfrac{2+2\cos\theta}{(\cos\theta+1)(\sin\theta)}$

$= \dfrac{2(1+\cos\theta)}{(\cos\theta+1)(\sin\theta)} = \dfrac{2}{\sin\theta} = \boxed{2\csc\theta}$ Whew!!

(14) $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{(1+\cos\theta)(\sin\theta)}$ *see #13

$= \dfrac{2+2\cos\theta}{(1+\cos\theta)(\sin\theta)} = \dfrac{2(1+\cos\theta)}{(1+\cos\theta)(\sin\theta)} = \dfrac{2}{\sin\theta} = \boxed{2\csc\theta}$

(15) $(\cos\theta+\sin\theta)^2 + (\cos\theta-\sin\theta)^2 = \cos^2\theta+2\cos\theta\sin\theta+\sin^2\theta+\cos^2\theta$
$-2\cos\theta\sin\theta+\sin^2\theta$

$(\cos\theta+\sin\theta)(\cos\theta+\sin\theta)\quad(\cos\theta-\sin\theta)(\cos\theta-\sin\theta)$

$= 1+1 = \boxed{2}$

(16) $(1+\tan\beta)^2 + (1-\tan\beta)^2 = 1+2\tan\beta+\tan^2\beta+1-2\tan\beta+\tan^2\beta$

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$= 1+\tan^2\beta+1+\tan^2\beta = \sec^2\beta+\sec^2\beta$

$= \boxed{2\sec^2\beta}$