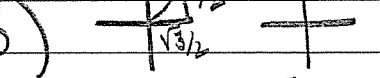
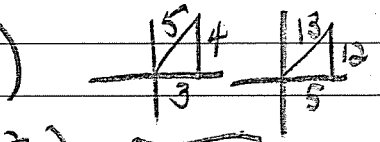
PW #2: Sum, Diff, Double, Half Angle Formulas

① $\sin\left(\sin^{-1}\left(\frac{1}{2}\right)+\cos^{-1}(0)\right)=\sin(\alpha+\beta)$



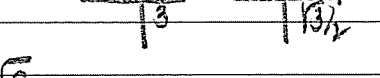
$\sin\alpha\cos\beta+\cos\alpha\sin\beta=\left(\frac{1}{2}\right)(0)+\left(\frac{\sqrt{3}}{2}\right)(1)=\boxed{\frac{\sqrt{3}}{2}}$

② $\cos\left(\tan^{-1}\left(\frac{4}{3}\right)+\cos^{-1}\left(\frac{5}{13}\right)\right)=\cos(\alpha+\beta)$



$\cos\alpha\cos\beta-\sin\alpha\sin\beta=\left(\frac{3}{5}\right)\left(\frac{5}{13}\right)-\left(\frac{4}{5}\right)\left(\frac{12}{13}\right)=\boxed{\frac{-33}{65}}$

③ $\cot\left(\sec^{-1}\left(\frac{5}{3}\right)+\frac{\pi}{6}\right)\overset{**}{=}\tan(\alpha+\beta)$



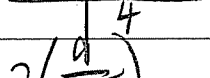
$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{\frac{4}{3}+\frac{\sqrt{3}}{3}}{1-\frac{4\sqrt{3}}{9}}=\frac{\frac{4+\sqrt{3}}{3}}{\frac{9-4\sqrt{3}}{9}}$

$=\frac{4+\sqrt{3}}{3}\cdot\frac{*3}{9-4\sqrt{3}}=\frac{12+3\sqrt{3}}{9-4\sqrt{3}}$ *Reciprocal for cot $\frac{9-4\sqrt{3}}{12+3\sqrt{3}}$

$\frac{9-4\sqrt{3}}{12+3\sqrt{3}}\cdot\frac{12-3\sqrt{3}}{12-3\sqrt{3}}=\frac{108-75\sqrt{3}+36}{144-27}=\frac{\cancel{144}^{48}-\cancel{75}^{25}\sqrt{3}}{\cancel{117}_{39}}$

$\boxed{\frac{48-25\sqrt{3}}{39}}$

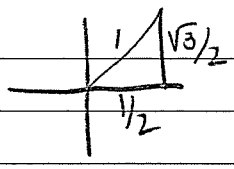
④ $\cos\left(2\sin^{-1}\left(\frac{3}{5}\right)\right)=\cos(2\theta)$ *3 choices for formula



$=1-2\sin^2\theta=1-2\left(\frac{3}{5}\right)^2=1-2\left(\frac{9}{25}\right)$

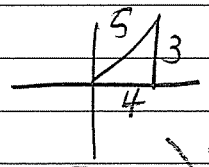
$=1-\frac{18}{25}=\boxed{\frac{7}{25}}$

(5) $\sin\left(2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) = \sin(2\theta)$



$= 2\sin\theta\cos\theta = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \boxed{\frac{\sqrt{3}}{2}}$

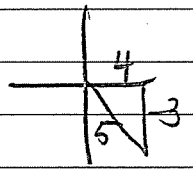
(6) $\cos^2\left(\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)\right) = \cos^2\left(\frac{\theta}{2}\right) =$



$= \left(\sqrt{\frac{1+\cos\theta}{2}}\right)^2 = \left(\sqrt{\frac{1+4/5}{2}}\right)^2 = \left(\sqrt{\frac{9/5}{2}}\right)^2$

$= \left(\sqrt{9/10}\right)^2 = \boxed{9/10}$

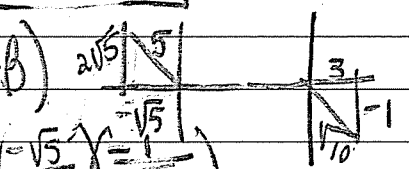
(7) $\csc\left(2\sin^{-1}\left(-3/5\right)\right) = \csc(2\theta) \overset{*}{=} \sin(2\theta)$



$= 2\sin\theta\cos\theta = 2\left(-3/5\right)\left(4/5\right) = -24/25$

Reciprocal → $\boxed{-25/24}$

(8) $\sin\left(\cos^{-1}\left(\frac{-\sqrt{5}}{5}\right) + \tan^{-1}\left(\frac{-1}{3}\right)\right) = \sin(\alpha+\beta)$



$\sin\alpha\cos\beta + \cos\alpha\sin\beta = \left(\frac{2\sqrt{5}}{5}\right)\left(\frac{3}{\sqrt{10}}\right) + \left(\frac{-\sqrt{5}}{5}\right)\left(\frac{-1}{\sqrt{10}}\right)$

$= \frac{6\sqrt{5}}{5\sqrt{10}} + \frac{\sqrt{5}}{5\sqrt{10}} = \frac{7\sqrt{5}}{5\sqrt{10}} = \frac{7}{5\sqrt{2}} = \boxed{\frac{7\sqrt{2}}{10}}$