PW #1: Simplify Trig Identities

(1) $1-\cos^2\theta = \boxed{\sin^2\theta}$

(2) $\csc^2\theta - 1 = \boxed{\cot^2\theta}$

(3) $\sin^2\theta - 1 = \boxed{-\cos^2\theta}$

(4) $1-\sec^2\theta = \boxed{-\tan^2\theta}$

(5) $\tan\alpha\cos\alpha = \frac{\sin\alpha}{\cos\alpha}\cdot\frac{\cos\alpha}{1} = \boxed{\sin\alpha}$

(6) $\cot\alpha\sin\alpha = \frac{\cos\alpha}{\sin\alpha}\cdot\frac{\sin\alpha}{1} = \boxed{\cos\alpha}$

(7) $\frac{\cos\alpha}{\sec\alpha} = \frac{\cos\alpha}{1}\cdot\frac{\cos\alpha}{1} = \boxed{\cos^2\alpha}$

(8) $\frac{\csc\alpha}{\sin\alpha} = \frac{1}{\sin\alpha}\cdot\frac{1}{\sin\alpha} = \frac{1}{\sin^2\alpha} = \boxed{\csc^2\alpha}$

(9) $\frac{\cot\alpha}{\tan\alpha} = \frac{\frac{\cos\alpha}{\sin\alpha}}{\frac{\sin\alpha}{\cos\alpha}} = \frac{\cos\alpha}{\sin\alpha}\cdot\frac{\cos\alpha}{\sin\alpha} = \frac{\cos^2\alpha}{\sin^2\alpha} = \boxed{\cot^2\alpha}$

(10) $\sin\alpha\sec\alpha = \frac{\sin\alpha}{1}\cdot\frac{1}{\cos\alpha} = \frac{\sin\alpha}{\cos\alpha} = \boxed{\tan\alpha}$

(11) $\cos\alpha\csc\alpha = \frac{\cos\alpha}{1}\cdot\frac{1}{\sin\alpha} = \boxed{\cot\alpha}$

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(12) $\dfrac{\tan\alpha}{\sin\alpha} = \dfrac{\frac{\sin\alpha}{\cos\alpha}}{\frac{\sin\alpha}{1}} = \dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{1}{\sin\alpha} = \dfrac{1}{\cos\alpha} = \boxed{\sec\alpha}$

(13) $\dfrac{\cos\alpha}{\cot\alpha} = \dfrac{\cos\alpha}{\frac{\cos\alpha}{\sin\alpha}} = \dfrac{\cos\alpha}{1}\cdot\dfrac{\sin\alpha}{\cos\alpha} = \boxed{\sin\alpha}$

(14) $(1-\sin\theta)(1+\sin\theta)$ FOIL $= 1-\sin^2\theta = \boxed{\cos^2\theta}$

(15) $(\sec\theta-1)(\sec\theta+1)$ FOIL $= \sec^2\theta - 1 = \boxed{\tan^2\theta}$

(16) $\cos\theta\csc\theta\tan\theta = \dfrac{\cos\theta}{1}\cdot\dfrac{1}{\sin\theta}\cdot\dfrac{\sin\theta}{\cos\theta} = \boxed{1}$

(17) $\cot\theta\sec\theta\sin\theta = \dfrac{\cos\theta}{\sin\theta}\cdot\dfrac{1}{\cos\theta}\cdot\dfrac{\sin\theta}{1} = \boxed{1}$

(18) $\dfrac{\sin\theta}{\csc\theta} + \dfrac{\cos\theta}{\sec\theta} = \dfrac{\sin\theta}{\frac{1}{\sin\theta}} + \dfrac{\cos\theta}{\frac{1}{\cos\theta}} = \dfrac{\sin\theta}{1}\cdot\dfrac{\sin\theta}{1} + \dfrac{\cos\theta}{1}\cdot\dfrac{\cos\theta}{1}$

$= \sin^2\theta + \cos^2\theta = \boxed{1}$

(19) $\dfrac{\csc\theta}{\sin\theta} - \dfrac{\cot\theta}{\tan\theta} = \dfrac{\frac{1}{\sin\theta}}{\sin\theta} - \dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}} = \dfrac{1}{\sin\theta}\cdot\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}\cdot\dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{1}{\sin^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1-\cos^2\theta}{\sin^2\theta} = \dfrac{\sin^2\theta}{\sin^2\theta} = \boxed{1}$

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(20) $\frac{\sec\theta}{\cos\theta} - 1 = \frac{\frac{1}{\cos\theta}}{\cos\theta} - 1 = \frac{1}{\cos\theta} \cdot \frac{1}{\cos\theta} - 1$

$= \frac{1}{\cos^2\theta} - 1 = \frac{1 - \cos^2\theta}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} = \boxed{\tan^2\theta}$

↖ get common denominators

(21) $\frac{\sec\theta}{\cos\theta} - \sec\theta\cos\theta = \frac{\frac{1}{\cos\theta}}{\cos\theta} - \frac{1}{\cos\theta} \cdot \frac{\cos\theta}{1}$

$= \frac{1}{\cos\theta} \cdot \frac{1}{\cos\theta} - 1 = \frac{1}{\cos^2\theta} - 1 = \frac{1 - \cos^2\theta}{\cos^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta} = \boxed{\tan^2\theta}$

(22) $\frac{1 + \tan^2\alpha}{\tan^2\alpha} = \frac{\sec^2\alpha}{\tan^2\alpha} = \frac{\frac{1}{\cos^2\alpha}}{\frac{\sin^2\alpha}{\cos^2\alpha}} = \frac{1}{\cos^2\alpha} \cdot \frac{\cos^2\alpha}{\sin^2\alpha}$

$= \frac{1}{\sin^2\alpha} = \boxed{\csc^2\alpha}$

* Don't cancel $\tan^2\alpha$!!

(23) $\frac{1 + \tan^2\alpha}{1 + \cot^2\alpha} = \frac{\sec^2\alpha}{\csc^2\alpha} = \frac{\frac{1}{\cos^2\alpha}}{\frac{1}{\sin^2\alpha}} = \frac{1}{\cos^2\alpha} \cdot \frac{\sin^2\alpha}{1}$

$= \frac{\sin^2\alpha}{\cos^2\alpha} = \boxed{\tan^2\alpha}$

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(24) $\cos^2 50^\circ + \sin^2 50 \Rightarrow \cos^2\theta + \sin^2\theta = \boxed{1}$

(25) $1 + \tan^2 50 \Rightarrow 1 + \tan^2\theta = \sec^2\theta \Rightarrow \boxed{\sec^2 50}$

(26) $\csc^2\alpha - \cot^2\alpha = \boxed{1}$

(27) $\sec^2 350^\circ - \tan^2 350^\circ \Rightarrow \sec^2\theta - \tan^2\theta$

$= \boxed{1}$