AP Stat Media Center Day #6: Multiple Choice Name Key
Ch. 7 & 8 and Free Response Practice
[89 total pts]

❖ Multiple Choice --- 3 pts each Select the best answer for each question.

Use this information to answer #1 & #2. A marketing survey compiled data on the number of cars in households. If X = the number of cars in a randomly selected household and we omit the rare cases of more than 5 cars, then X has the following probability distribution:

X	0	1	2	3	4	5
P(X)	0.24	0.37	0.20	0.11	0.05	0.03

D 1. What is the probability that a randomly chosen household has at least two cars?
A. 0.19 B. 0.20 C. 0.29 D. 0.39 E. 0.61 $P(X \geq 2) = .2 + .11 + .05 + .03$

2. What is the expected value of the number of cars in a randomly selected household?
C A. 2.5 B. 0.1667 C. 1.45 D. 1 E. cannot be determined
$E(X) = (0)(.24) + (1)(.37) + (2)(.2) + \ldots + (5)(.03) = 1.45$

3. In the town of Lakeville, the number of cell phones in a household is a random variable W with the following distribution:

X	0	1	2	3	4	5
P(X)	0.1	0.1	0.25	0.3	0.2	0.05

A The variance of the number of cell phones in a randomly selected household is
A. 1.745 B. 1.87 C. 2.55 D. 1.7078 E. 1.32
$\sigma_x^2 = (0-2.55)^2(.1) + \ldots + (5-2.55)^2(.05) = 1.7449$

4. A random variable Y has the following probability distribution:

X	-1	0	1	2
P(X)	4C	2C	0.07	0.03

$4C + 2C + .1 = 1$

B The value of the constant C is $6C = .9$
A. 0.10 B. 0.15 C. 0.20 D. 0.25 E. 0.75
$C = .15$

5. The variance of the sum of two random variables X and Y is
D A. $\sigma_x + \sigma_y$ B. $(\sigma_x)^2 + (\sigma_y)^2$ C. $\sigma_x + \sigma_y$, but only if X and Y are independent
D. $(\sigma_x)^2 + (\sigma_y)^2$, but only if X and Y are independent E. None of these

6. For which of the following counts would a binomial probability model be reasonable?
A. The number of traffic tickets written by each police officer in a large city during one month. n not defined
C B. The number of hearts in a hand of five cards dealt from a standard deck of 52 cards that has been thoroughly shuffled. p does not remain the same 13/52, 12/51, ...
C. The number of 7's in a randomly selected set of five single digits from a table of random digits.
D. The number of phone calls received in a one-hour period. n not defined
E. All of the above.

7. It is known that 90% of the widgets made by Buckley Industries meet specifications. Every hour a sample of 18 widgets is selected at random for testing and the number of widgets that meet specifications is recorded. What is the approximate mean and standard deviation of the number of widgets meeting specifications?

Binomial · n = 18 · S = meets, F = not meets · Independent · p = .9 q = .10

D

A. $\mu = 1.62; \sigma = 1.414$ B. $\mu = 1.62; \sigma = 1.265$ C. $\mu = 16.2; \sigma = 1.62$

D. $\mu = 16.2; \sigma = 1.273$ E. $\mu = 16.2; \sigma = 4.025$

$\mu = np = 18(.9) = 16.2 \quad \sigma_x = \sqrt{npq} = \sqrt{(18)(.9)(.1)} = \sqrt{1.62} = 1.273$

8. A raffle sells tickets for \$10 and offers a prize of \$500, \$1000, or \$2000. Let C be a random variable that represents the monetary prize in the raffle drawing. The probability distribution of C is given below.

X	\$0	\$500	\$1000	\$2000
P(X)	0.60	0.05	0.13	0.22

C

The expected ***profit* for ticket holder** when playing the raffle is

A. \$595 B. \$865 C. \$585 D. \$635 E. \$485

$E(x) = (0)(.6) + (500)(.05) + (1000)(.13) + (2000)(.22) = 595 - 10 = 585$

9. To pass the time, a toll booth collector counts the number of cars that pass through his booth until he encounters a driver with red hair. Suppose we define the random variable Y = the number of cars the collector counts until he gets a red-headed driver for the first time. Is Y a geometric random variable?

A

T A. Yes - all conditions for the geometric setting are met.

F B. No - "red-headed driver" and "non-red-headed driver" are not the same as "success" and "failure".

F C. No - we can't assume that each "trial" (that is, each car) is independent of previous trials.

F D. No - the number of trials is not fixed.

F E. No - the probability of a driver being red-headed is not the same for each trial.

Use the following table to answer #10 - #12. Students are classified by television usage (unending, average, and infrequent) and how often they exercise (regular, occasional, and never), resulting in the following probability table.

TV usage	Exercise: Regular (Y = 1)	Occasional (Y = 2)	Never (Y = 3)	
Unending (X = 1)	.05	.05	.10	.2
Average (X = 2)	.20	.15	.10	.45
Infrequent (X = 3)	.15	.15	.05	.35
	.4	.35	.25	

10. What is the probability distribution for X?

C

A. P(X = 1) = 0.05, P(X = 2) = 0.2, P(X = 3) = 0.15

B. P(X = 1) = 0.10, P(X = 2) = 035, P(X = 3) = 0.30

C. P(X = 1) = 0.20, P(X = 2) = 0.45, P(X = 3) = 0.35

D. P(X = 1) = 0.40, P(X = 2) = 0.35, P(X = 3) = 0.25

E. It cannot be determined from the given information.

11. What is the probability P(X = 2, Y = 3), that is, the probability that a students has average television usage but never exercises? $P(A \cap B) = .10$

A

A. 0.10 B. 0.1125 C. 0.22 D. 0.40 E. 0.60

12. What is the probability P(X = 2|Y = 3), that is, the probability that a student has average television usage given that he never exercises? $P(A|B) = P(A \cap B)/P(B) = \frac{.1}{.25} = .4$

D

A. 0.10 B. 0.17 C. 0.22 D. 0.40 E. 0.60

13. Are X and Y independent?

A. Yes, because the conditional probabilities $P(X = x|Y = y)$ equal the corresponding unconditional probability $P(X = x)$.

B. Yes, because the joint probabilities are equal to the product of the respective probabilities.

C. Yes, because of either of the above answers.

D D. No.

E. The answer cannot be determined from the given information.

TV usage	Exercise: Regular (Y = 1)	Exercise: Occasional (Y = 2)	Exercise: Never (Y = 3)
Unending (X = 1)	.05	.05	.10
Average (X = 2)	.20	.15	.10
Infrequent (X = 3)	.15	.15	.05

14. Following are parts of the probability distributions for the random variables X and Y. If X and Y are independent and the joint probability $P(X = 1, Y = 2) = 0.1$, what is $P(X = 4)$?

C A. 0.10 B. 0.20 C. 0.30 D. 0.40 E. 0.50

$P(A \cap B) = P(A) \cdot P(B)$

$.1 = x \cdot .5$

$.2 = x$

$P(X=1) = .2$

$P(X=4) = 1 - .7 = .3$

x	$P(x)$	y	$P(y)$
1	.2?	1	.4
2	.2	2	?
3	.3	3	.1
4	?		

15. Suppose X and Y are random variables with $E(X) = 25$, $\mathrm{var}(X) = 3$, $E(Y) = 30$, $\mathrm{var}(Y) = 4$. What are the expected value and variance of the random variable $X + Y$?

E A. $E(X+Y) = 55, \mathrm{var}(X+Y) = 3.5$ B. $E(X+Y) = 55, \mathrm{var}(X+Y) = 5$

C. $E(X+Y) = 55, \mathrm{var}(X+Y) = 7$ D. $E(X+Y) = 27.5, \mathrm{var}(X+Y) = 7$

E. There is insufficient information to answer this question.

Not told independent

16. Suppose X and Y are random variables with $\mu_x = 10, \sigma_x = 3, \mu_y = 15, and\ \sigma_y = 4$. Given that X and Y are independent, what are the mean and standard deviation of the random variable $X + Y$?

B A. $\mu_{x+y} = 25, \sigma_{x+y} = 3.5$ B. $\mu_{x+y} = 25, \sigma_{x+y} = 5$ C. $\mu_{x+y} = 25, \sigma_{x+y} = 7$

D. $\mu_{x+y} = 12.5, \sigma_{x+y} = 7$ E. There is insufficient information to answer this question.

$\mu_{x+y} = \mu_x + \mu_y = 10 + 15 = 25$ $\sigma_{x+y} = \sqrt{\sigma_x^2 + \sigma_y^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

17. Suppose X and Y are random variables with $E(X) = 500$, $\mathrm{var}(X) = 50$, $E(Y) = 400$, $\mathrm{var}(Y) = 30$. Given that X and Y are independent, what are the expected value and variance of the random variable $X - Y$?

B A. $E(X-Y) = 100, \mathrm{var}(X-Y) = 20$ B. $E(X-Y) = 100, \mathrm{var}(X-Y) = 80$

C. $E(X-Y) = 900, \mathrm{var}(X-Y) = 20$ D. $E(X-Y) = 900, \mathrm{var}(X-Y) = 80$

E. There is insufficient information to answer this question.

$\mu_{x-y} = \mu_x - \mu_y = 500 - 400 = 100$ $\sigma^2_{x-y} = \sigma^2_x + \sigma^2_y = 50 + 30 = 80$

18. Following are parts of the probability distributions for the random variables X and Y. If X and Y are independent and two joint probabilities are $P(X = 1, Y = 1) = 0.025$ and $P(X = 3, Y = 3) = 0.08$, what is $P(X = 2, Y = 2)$?

$.08 = x \cdot .2$

$.4 = P(X=3)$

$(.5)(.55) = .275$

x	$P(x)$	y	$P(y)$
1	.1	1	?
2	.5 ?	2	?
3	.4?	3	.2

D A. 0.115 B. 0.725 C. 0.333 D. 0.275 E. 0.895

19. Suppose X and Y are random variables with $\mu_x = 720, \sigma_x = 6, \mu_y = 240, and\ \sigma_y = 8$. Given that X and Y are independent, what are the mean and standard deviation of the random variable $X+Y$?

$\mu_{x+y} = \mu_x + \mu_y = 720 + 240 = 960$ $\sigma_{x+y} = \sqrt{6^2+8^2} = 10$

B

A. $\mu_{x+y} = 960, \sigma_{x+y} = 7$ B. $\mu_{x+y} = 960, \sigma_{x+y} = 10$ C. $\mu_{x+y} = 960, \sigma_{x+y} = 14$

D. $\mu_{x+y} = 960, \sigma_{x+y} = 100$ E. There is insufficient information to answer this question.

❖ Free Response --- 4 pts each

20. Willie's Dawgbacker Tours offers a popular half-day trip to various UGA attractions. There must be at least 2 passengers for the trip to run, and the vehicle will hold up to 6 passengers. The random variable X is defined as the number of passengers on Willie's tour on a randomly selected day.

Passengers x_i	2	3	4	5	6
Probability p_i	0.15	0.25	0.35	0.20	0.05

a) On average, what is the expected number of passengers Willie should plan to take on his tour each day? **Show work.** $E(X) = (2)(.15) + (3)(.25) + \ldots + (6)(.05) = 3.75$

b) For any randomly selected day, on average, how much will the number of actual passengers on Willie's tour deviate from the number of passengers he expected? **Show work.**
[HINT: This is called standard deviation.]

$\sigma_X = \sqrt{(2-3.75)^2(.15) + \ldots + (6-3.75)^2(.05)} = 1.09$

c) Willie plans to charge each passenger $150 for the tour. It costs him $100 for gas, parking permits, and attraction tickets for each trip. Define random variable C in terms of X describing the amount of profit Willie will get on a randomly selected day.

$C = 150X - 100$

d) On average, how much should Willie expect to collect from his tour on any randomly selected day? **Show work.** $\mu_C = \mu_{150X-100} = 150(\mu_X) - 100 = 150(3.75) - 100 = \$462.50$

e) For any randomly selected day, on average how much will the actual amount Willie brings in for his tour deviate from amount he expected? **Show work.** $

$\sigma_C = \sigma_{150X-100} = 150(\sigma_X) = 150(1.09) = \$163.50$

f) Willie's cousin Jay runs a similar type of touring van around Athens. Define Y as the number of passengers on Jay's tour on a randomly selected day. Assume these events are independent. Define $T = X + Y$. How many total passengers can Willie and Jay expect on a randomly selected day?

Passengers y_i	2	3	4	5
Probability p_i	0.3	0.4	0.2	0.1

$\mu_Y = (2)(.3) + \ldots + (5)(.1) = 3.1$

$\mu_T = \mu_{X+Y} = \mu_X + \mu_Y = 3.75 + 3.1 = 6.85$

g) What is the variance of T? **Show work.** $\sigma_T^2 = \sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 = (1.09)^2 + (.9434)^2 = 2.08$

h) Jay also charges $150 per passenger, but only spends $80 on gas, parking permits, and attraction tickets for each trip. On average, how much profit should Willie and Jay expect to bring in together on a randomly selected day. **Show work.**

$D = 150(Y) - 80$ $\mu_{C+D} = \mu_C + \mu_D = 462.50 - 385 = \$847.50$

$\mu_D = 150(3.1) - 80 = 385$