Answers   Inference Practice Worksheet #3

1. a) State: We will create a 99% confidence interval for a 1-sample proportion to estimate the true population proportion of shoppers at a specific mall who spend more than $25.

Plan: Parameter: *p* = the true population proportion of shoppers at a specific mall who spend more than $25

Random: The problem states a simple random sample of shoppers was selected.

Independence: population of all shoppers at this mall $\geq$ 10(550) Condition met for independence

Large Counts: np $\geq$10 352 $\geq$10 nq $\geq$ 10 198 $\geq$ 10 Sample size large enough to consider approximately Normal.

Do:

$$\hat{p} = 0.64 \quad \hat{q} = 0.36 \quad n = 550 \quad se = \sqrt{\frac{(.64)(.36)}{550}} \quad z^* = 2.576 \;\; CL = .99$$

$$.64 \pm 2.576\left(\sqrt{\frac{(.64)(.36)}{550}}\right) \quad (0.587, 0.693)$$

We are 99% confident the true population proportion of shoppers at this mall who spent more than $25 is in the interval 58.7% to 69.3% for a sample size of 550 shoppers.

2. a) State: We will create a 95% confidence interval for a 1-sample proportion to estimate the true population proportion of machine parts damaged in shipment.

Plan: Parameter: *p* = the true population proportion of machine parts damaged in shipment

Random: The problem states a random sample of machine parts was selected.

Independence: population of all machine parts in shipment $\geq$ 10(225) Condition met for independence

Large Counts: np $\geq$10 18 $\geq$10 nq $\geq$ 10 207 $\geq$ 10 Sample size large enough to consider approximately Normal.

Do:

$$\hat{p} = 0.08 \quad \hat{q} = 0.92 \quad n = 225 \quad se = \sqrt{\frac{(.08)(.92)}{225}} \quad z^* = 1.96 \;\; CL = .95$$

$$.08 \pm 1.96\left(\sqrt{\frac{(.08)(.92)}{225}}\right) \quad (0.0445, 0.1155)$$

We are 99% confident the true population proportion of machine parts damaged in the shipment is in the interval 4.5% to 11.6% for a sample size of 225 machine parts.

3. a) State: We will create a 95% confidence interval for a 1-sample mean to estimate the true population mean number of ounces a bottling machine inserts into each bottle it fills.

Plan: Parameter: *μ* = the true population mean number of ounces a bottling machine inserts into each bottle it fills.

Random: The problem states a random sample of bottles was selected.

Independence: population of all bottles $\geq$ 10(36) Condition met for independence

Large Counts: n = 36 36 $\geq$ 30 CLT states that the sample size is large enough to assume approximate Normal distribution.

Do: $\bar{x} = 16.1 \quad s = 0.12 \quad n = 36 \quad df = 35 \quad CL = .95 \quad t^* = 2.042 \quad 16.1 \pm 2.042\left(\frac{0.12}{\sqrt{36}}\right) \quad (16.093,\ 16.107)$

We are 95% confident the true population mean number of ounces placed in bottles by a machine is in the interval 16.093 oz and 16.107 oz for a sample size of 36 bottles.

b) $\bar{x} = 16.1 \quad s = 0.12 \quad n = 36 \quad df = 35 \quad CL = .99 \quad t^* = 2.75 \quad 16.1 \pm 2.75\left(\frac{0.12}{\sqrt{36}}\right) \quad (16.091,\ 16.109)$

We are 99% confident the true population mean number of ounces placed in bottles by a machine is in the interval 16.091 oz and 16.109 oz for a sample size of 36 bottles.

c) The confidence interval at 95% is smaller than the confidence interval at 99%. The critical value, 2.042, at 95% will give a smaller margin of error than the critical value, 2.75, at 99%. The larger confidence level results in a larger, less specific interval.

4. a) State: With what confidence level can it be asserted that 83.2% ± 3% of adults in the U.S. support the decision to go to war in Iraq?

Plan: Parameter: *p* = the true population proportion of adults in the U.S. who support going to war in Iraq

Random: The problem does not state a random sample adults was surveyed. No reason to assume the adults selected in the telephone survey were not representative of the population of all U.S. adults.

Independence: population of all U.S. adults $\geq$ 10(1000) Condition met for independence

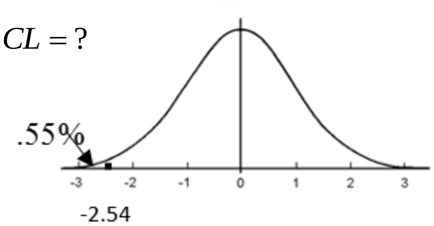
Large Counts: np $\geq$ 10 832 $\geq$10 nq $\geq$ 10 168 $\geq$ 10 Sample size large enough to consider approximately Normal.

Do:

$\hat{p} = 0.832 \quad \hat{q} = 0.168 \quad n = 1000 \quad se = \sqrt{\frac{(.832)(.168)}{100}} = 0.0118 \quad z^* = ? \quad CL = ?$

$\pm .08 = z^*(0.0118)$

$\pm 2.54 = z^* \qquad P(z < -2.54) = 0.0055$



0.55% lies in <u>EACH</u> tail, so the confidence interval is 98.89%. Therefore the confidence interval for 83.2% ± 3% is a 98.89% confidence interval estimate for the true population proportion of all U.S. adults who support the war in Iraq for a sample size of 1000 U.S. adults.

b) 83.2% + 3% = 86.2% (0.862)(191 million) = 165 million
83.2% - 3% = 81.2% (0.812)(191 million) = 153 million

We are 98.89% confident that the true population number of U.S. adults who support the war in Iraq is in the interval 153 million to 165 million for a sample size of 1000 U.S. adults.

5. a) State: We will create a 90% confidence interval for a 1-sample proportion to estimate the true population proportion of unemployed men in the U.S. who are married.

Plan: Parameter p = the true population proportion of men in the U.S. who are not married

Random: The problem does not state a random sample of U.S. unemployed men was surveyed. No reason to assume the men selected were not representative of the population of all unemployed U.S. males.

Independence: population of all unemployed U.S. males $\geq$ 10(3499) Condition met for independence

Large Counts: np $\geq$ 10 1356 $\geq$10 nq $\geq$ 10 2143 $\geq$ 10 Sample size large enough to consider approximately Normal.

Do:

$$\hat{p} = 0.3875 \quad \hat{q} = 0.6125 \quad n = 3499 \quad se = \sqrt{\frac{(.3875)(.6125)}{3499}} \quad z^* = 1.645 \quad CL = .90$$

$$.3875 \pm 1.645\left(\sqrt{\frac{(.3875)(.6125)}{3455}}\right) \qquad (0.37399, 0.40109)$$

We are 90% confident the true population proportion of unemployed males who are not married is in the interval 37.4% to 40.1% for a sample size of 3455 unemployed males.

6. a) State: We will create a 90% confidence interval for a 1-sample mean to estimate the true population mean life expectancy for batteries at a specific manufacturing plant.

Plan: Parameter: μ = the true population mean life expectancy for batteries at a specific manufacturing plant

Random: The problem states a random sample of batteries was selected.

Independence: population of all batteries produced at this plant $\geq$ 10(64) Condition met for independence

Large Counts: n = 64 64 $\geq$ 30 CLT states that the sample size is large enough to assume approximate Normal distribution.

Do:

$$\bar{x} = 12.35 \quad s = 2.4 \quad n = 64 \quad df = 63 \quad CL = .90 \quad t^* = 1.671 \quad 12.35 \pm 1.671\left(\frac{2.4}{\sqrt{64}}\right) \quad (12.287,\ 12.413)$$

We are 90% confident the true population mean life expectancy for batteries at a particular manufacturing plant is in the interval 12.287 months and 12.413 months for a sample size of 64 batteries.

b) State: We will create a 90% confidence interval for a 1-sample mean to estimate the true population mean life expectancy for batteries at a specific manufacturing plant.

Plan: Parameter: μ = the true population mean life expectancy for batteries at a specific manufacturing plant

Random: The problem states a random sample of batteries was selected.

Independence: population of all batteries produced at this plant ≥ 10(100) Condition met for independence

1

Large Counts: n = 100 100 ≥ 30 CLT states that the sample size is large enough to assume approximate Normal distribution.

Do:

$\bar{x} = 12.35 \quad s = 2.4 \quad n = 100 \quad df = 99 \quad CL = .90 \quad t^* = 1.66 \quad 12.35 \pm 1.66\left(\frac{2.4}{\sqrt{64}}\right) \quad (11.952,\ 12.748)$

We are 90% confident the true population mean life expectancy for batteries at a particular manufacturing plant is in the interval 11.952 months and 12.748 months for a sample size of 100 batteries.

c) Increased sample size shortens the confidence interval. Margin of error is smaller.

7. Consider how large $\sqrt{p(1-p)}$ can be:

p:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\sqrt{p(1-p)}$:	0.3	0.4	0.458	0.4899	0.5	0.4899	0.458	0.4	0.3

Therefore $\sqrt{p(1-p)}$ has the largest value when *p* = 0.5. Therefore *se* is largest at $\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.5)(0.5)}{n}}$.

This gives the largest margin of error.

$z^*(se) \le me \qquad Conf\ Level = .96 \qquad z^* = 2.054$

$2.054\left(\sqrt{\frac{(0.5)(0.5)}{n}}\right) \le \pm 0.03 \qquad \frac{2.054(0.5)}{\sqrt{n}} \le \pm 0.03 \qquad 2.054(0.5) \le \pm 0.03\sqrt{n}$

$\frac{2.054(0.5)}{0.03} \le \sqrt{n} \qquad \sqrt{n} \ge 34.233 \qquad n \ge 1171$

One would need a sample size of at least 1171 fish to obtain a margin of error of ±3% at a 96% confidence level.

8. a)

$z^*(se) \le me \qquad Conf\ Level = .99 \qquad z^* = 2.58$

$2.58\left(\sqrt{\frac{(0.5)(0.5)}{n}}\right) \le \pm 0.06 \qquad \frac{2.58(0.5)}{\sqrt{n}} \le \pm 0.06 \qquad 2.58(0.5) \le \pm 0.06\sqrt{n}$

$\frac{2.58(0.5)}{0.06} \le \sqrt{n} \qquad \sqrt{n} \ge 21.5 \qquad n \ge 463$

One would need a sample size of at least 463 executives to obtain a margin of error of ±6% at a 99% confidence level.

b)

$$z^*(se) \le me \qquad Conf\ Level = .99 \qquad z^* = 2.58$$

$$2.58\left(\sqrt{\frac{(0.5)(0.5)}{n}}\right) \le \pm 0.03 \qquad \frac{2.58(0.5)}{\sqrt{n}} \le \pm 0.03 \qquad 2.58(0.5) \le \pm 0.03\sqrt{n}$$

$$\frac{2.58(0.5)}{0.03} \le \sqrt{n} \qquad \sqrt{n} \ge 43 \qquad n \ge 1849$$

One would need a sample size of at least 1849 executives to obtain a margin of error of ±3% at a 99% confidence level.

c)

$$z^*(se) \le me \qquad Conf\ Level = .99 \qquad z^* = 2.58$$

$$2.58\left(\sqrt{\frac{(0.5)(0.5)}{n}}\right) \le \pm 0.02 \qquad \frac{2.58(0.5)}{\sqrt{n}} \le \pm 0.02 \qquad 2.58(0.5) \le \pm 0.02\sqrt{n}$$

$$\frac{2.58(0.5)}{0.02} \le \sqrt{n} \qquad \sqrt{n} \ge 64.5 \qquad n \ge 4161$$

One would need a sample size of at least 4161 executives to obtain a margin of error of ±2% at a 99% confidence level.

d) $me = 2.58\left(\frac{1}{2}\right)\sqrt{\frac{(.5)(.5)}{n}} \qquad me = 2.58\sqrt{\frac{(.5)(.5)}{4n}}$

In order to cut a confidence interval in half, you would need to quadruple the sample size as seen in parts a) and b). The sample size of approximately 463 executives gave a margin of error of 6% and a sample size of approximately 1849 executives gave a margin of error of 3%.

To reduce a confidence interval to a third of the origin, the sample size would need to increase ninefold as seen in parts a) and c). The sample size of approximately 463 executives gave a margin of error of 6% and a sample size of approximately 4161 executives gave a margin of error of 2%.