Practice Handout: Chi-Square Tests p. 1

Follow your Chi-Square Template

(1) • p_1 = proportion of jurors 21 to 40 yrs old

p_2 = " " " 41 to 50 yrs old

p_3 = " " " 51 to 60 yrs old

p_4 = " " " over 60 yrs old

• χ^2 - test for Goodness of Fit

• Conditions:

(1) SRS - stated in problem

(2) exp count ≥ 1 80% of exp count ≥ 5 Conditions met

See Table below

Age:	21 to 40	41 to 50	51 to 60	over 60	Total
Obs:	5	9	19	33	66
Exp:	27.72	15.18	10.56	12.54	

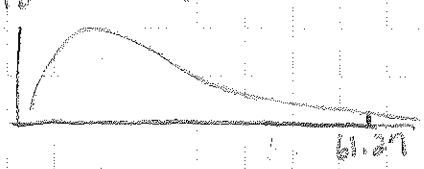
• Hypotheses:

H_0: $p_1 = .42$ $p_2 = .23$ $p_3 = .16$ $p_4 = .19$

Ha: at least one of these proportions is different

• Calculations: $\chi^2 = \frac{(5-27.72)^2}{27.72} + \frac{(9-15.18)^2}{15.18} + \frac{(19-10.56)^2}{10.56} + \frac{(33-12.54)^2}{12.54} = 61.27$

df = 3 α = .05



$P(\chi^2 \geq 61.27) = .0000$

Since our p-value of .000 is smaller than our significance level (or any significance level) of $\alpha = .05$, our data is significant! We have evidence to reject the null. We can conclude the juries are not selected at random.

② • H_0: attending preschool and pre-algebra achievement are independent
(or there is no association between attending preschool and prealgebra achievement)

H_a: attending preschool and pre-algebra achievement are not independent
(or there is ~~a~~ association between attending pre-school and prealgebra achievement)

• χ^2 - test for Independence
• Conditions:
① SRS - stated
② exp counts ≥ 1 80% exp counts ≥ 5 Conditions Met
See Table Below

	Below Grade Level	At Grade Level	Advanced
Preschool	8 (5.6)*	6 (8.4)	6 (6)
No Preschool	6 (8.4)	15 (12.6)	9 (9)

* Recall: Exp Val = (row total)(column total)/grand total
= (20)(14)/50 = 5.6

• Calculations: $\chi^2 = \frac{(8-5.6)^2}{5.6} + \frac{(6-8.4)^2}{8.4} + \ldots \frac{(9-9)^2}{9} = 2.86$

$\alpha = .05$ $df = 2$
$(3-1)(2-1)$ $P(\chi^2 \geq 2.86) = .2397$

Since our p-value of .2397 is greater than our significance level of $\alpha = .05$, it is significant. We have evidence to fail to reject the null. We can conclude there is no association between a child attending preschool and his prealgebra achievement.

(3) p_1 = true proportion of students using a new math textbook who perform below grade level on a state math test

p_2 = true proportion of students using a new math textbook who perform on grade level on a state math test

p_3 = true proportion of students using a new math textbook who perform above grade level on a state math test

p_4 = true proportion of students using an old math textbook who perform below grade level on a state math test

p_5 = true proportion of students using an old math textbook who perform on grade level on a state math test

p_6 = true proportion of students using an old math textbook who perform above grade level on a state math test

χ^2-test for homogeneity

Conditions: (1) SRS - random sample, no reason to assume otherwise

(2) exp counts ≥ 1 (see table) Condition met

80% of exp counts ≥ 5 (see table) Condition met

	Below Grade Level	At Grade Level	Advanced
New textbook	7 (4.8)	8 (10)	5 (5.2)
Old textbook	5 (7.2)	17 (15)	8 (7.8)

Hypothesis: H_0: $p_1 = p_2 = p_3 = p_4 = p_5 = p_6$

H_a: $p_1 \neq p_2 \neq p_3 \neq p_4 \neq p_5 \neq p_6$

H_0: there is no difference in the proportion of students who perform below level, on level, or above level using new or old textbooks

H_a: there is a difference in the proportion of students who perform below level, on level, or above level using new or old textbooks

Calculations: $\chi^2 = \frac{(7-4.8)^2}{4.8} + \frac{(8-10)^2}{10} + \frac{(5-5.2)^2}{5.2} + \frac{(5-7.2)^2}{7.2}$

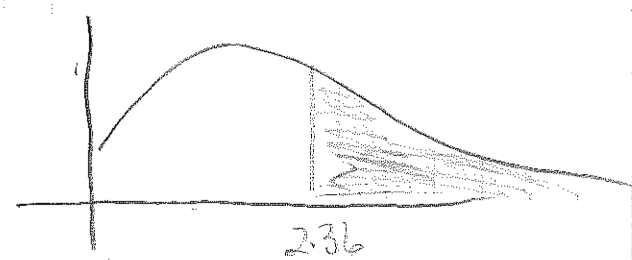
$\alpha = .05$ $+ \frac{(17-15)^2}{15} + \frac{(8-7.8)^2}{7.8} = 2.36$

$df = 2$

Components for χ^2: 1.008 0.4 0.008

** Found using $L_1, L_2, + L_3$ 0.672 0.2667 0.0051

$P(\chi^2 \geq 2.36) = .307$



Since our p-value of .307 is greater than our significance level $\alpha = .05$, we have evidence to fail to reject the null. We can conclude there is no difference in the proportion of students who perform below, on, or above grade level using new or old math textbooks.

(4) χ^2-test for Goodness of Fit

p_1 = proportion of cars that are white

p_2 = proportion of cars that are blue

p_3 = proportion of cars that are red

p_4 = proportion of cars that are black

p_5 = proportion of cars that are other colors

Conditions: (1) SRS - stated

(2) exp counts ≥ 1 (see table) Condition Met

80% of exp counts ≥ 5 (see table) Condition Met

	White	Blue	Red	Black	Other
Observed:	140	100	270	230	90
Expected:	(124.5)	(124.5)	(290.5)	(249)	(41.5)

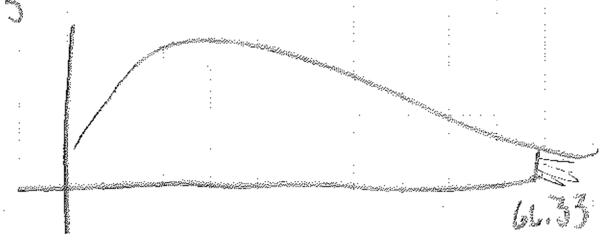
Hypothesis: H_0: $p_1 = .15$ $p_2 = .15$ $p_3 = .35$ $p_4 = .3$ $p_5 = .05$

H_a: at least one of these proportions is different

Calculation: $\chi^2 = \frac{(140-124.5)^2}{124.5} + \frac{(100-124.5)^2}{124.5} + \frac{(270-290.5)^2}{290.5} +$

df = 4 $\alpha = .05$ $\frac{(230-249)^2}{249} + \frac{(90-41.5)^2}{41.5} = 66.33$

$P(\chi^2 \geq 66.33) = .0000$



Since our p-value of .0000 is less than our significance level of .05, we have evidence to reject the null. We can conclude at least one of these proportions is different. The color of a car does influence the chance it will be stolen.

(5) p_1 = proportion of viewers watching channel 2

p_2 = proportion of viewers watching channel 3

p_3 = proportion of viewers watching channel 4

p_4 = proportion of viewers watching channel 5

χ^2 test for Goodness of Fit

Conditions: (1) SRS - no reason to assume otherwise

(2) exp counts ≥ 1 (see table) Condition met

80% exp counts ≥ 5 (see table) Condition met

	Ch 2	Ch 3	Ch 4	Ch 5
Observed:	139	138	(112)	(111)
Expected:	(150)	(125)	(100)	(125)

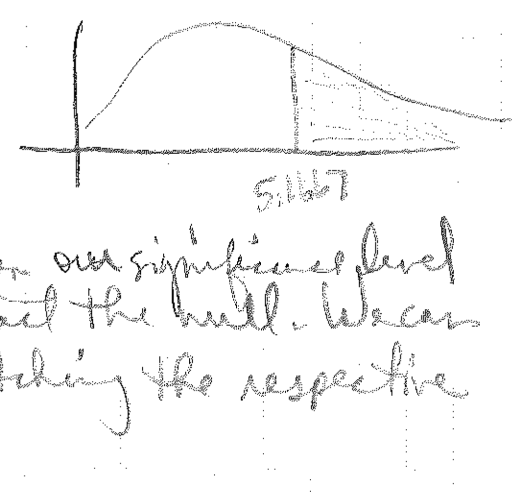
Hypothesis: $H_0: p_1 = .3 \quad p_2 = .25 \quad p_3 = .2 \quad p_4 = .25$

H_a: at least one of these proportions is different

Calculations: $\chi^2 = \frac{(139-150)^2}{150} + \frac{(138-125)^2}{125} + \frac{(112-100)^2}{100} + \frac{(111-125)^2}{125}$

df = 3 $\alpha = .05$ $= 5.1667$

$P(\chi^2 \geq 5.1667) = .1599$



Since our p-value of .1599 is greater than our significance level $\alpha = .05$, we have evidence to fail to reject the null. We can conclude the proportions of viewers watching the respective stations is as stated.

⑥ H_0: ~~Having~~ been abused as a child and carrying a certain abnormal gene is not associated with committing violent crime

H_a: having been abused as a child and carrying a certain abnormal gene is associated with committing violent crimes.

- χ^2 test for Independence
- Conditions: SRS - no reason to think otherwise
  exp counts ≥ 1 80% of exp counts ≥ 5 $\boxed{\text{Met}}$

	Not Abused	Abused, Normal Gene	Not Abused Abnormal Gene	Abused, Abnormal Gene
Criminal Behavior	48 (55)	21 (22.1)	32 (33.1)	26 (16.8)
Normal Behavior	201 (194)	79 (77.9)	118 (116.9)	50 (59.2)

- Calculations: $\chi^2 = \frac{(48-55)^2}{55} + \frac{(21-22.1)^2}{22.1} + \ldots + \frac{(50-59.2)^2}{59.2} = 7.72$

$df=3$ $\alpha = .05$ $P(\chi^2 \geq 7.75) = .0514$

Since our p-value of .0514 is greater than our significance level of .05, we have evidence to fail to reject the null. We can conclude there is no association between abuse, carrying an abnormal gene and participating in criminal behavior.

(7) • p_1 = proportion of 1st species of fruit flies
p_2 = proportion of 2nd species of fruit flies
p_3 = proportion of 3rd species of fruit flies
p_4 = proportion of 4th species of fruit flies

• χ^2 test of Goodness of Fit

• Conditions: SRS - no reason to think otherwise

exp counts ≥ 1 80% of exp counts ≥ 5 $\boxed{\text{met}}$

Fruit Flies	1st Species	2nd Species	3rd Species	4th Species
Obs	226	764	733	2277
Exp	$1/16(4000) = 250$	$3/16(4000) = 750$	$3/16(4000) = 750$	$9/16(4000) = 2250$

• Hypothesis:
H_0: $p_1 = 1/16$ $p_2 = 3/16$ $p_3 = 3/16$ $p_4 = 9/16$

H_a: the actual proportions are different from those claimed

Calculation:
df = 3 $\alpha = .10$ $\chi^2 = \frac{(226-250)^2}{250} + \frac{(764-750)^2}{750} + \frac{(733-750)^2}{750} + \frac{(2277-2250)^2}{2250}$

$P(\chi^2 \geq 3.275) = .3511$

Since our p-value of .3511 is greater than our significance level $\alpha = .10$ we have evidence to fail to reject the null. We can conclude the true proportions of fruit flies are not different from those claimed