Problem Set #1 Unit 1: Central Limit Theorem

(1) sampling distribution of sample means

(2) sampling error

(3) population mean

(4) standard error of the mean $\frac{\sigma}{\sqrt{n}}$

(5) normal $n \geq 30$

(6) $z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

(7) $\mu = 17.2 \quad \sigma = 2.5 \quad n = 55$

$$P(17 < x < 18) = P\left(\frac{17 - 17.2}{\frac{2.5}{\sqrt{55}}} < z < \frac{18 - 17.2}{\frac{2.5}{\sqrt{55}}}\right)$$

$$= P(-.593 < z < 2.37) = \boxed{.715}$$

(8) $\mu = \$186.80 \quad n = 50 \quad \sigma = 32$

$$P(\bar{x} < 175) = P\left(z < \frac{175 - 186.8}{\frac{32}{\sqrt{50}}}\right)$$

$$= P(z < -2.61) = .005$$

(9) a) $\mu = 52,174 \quad \sigma = 7500 \quad n = 1$

$$P(\bar{X} < 50000) = P\left(z < \frac{50000 - 52174}{7500}\right)$$

$$= P(z < -.29) = \boxed{.386}$$

b) $\mu = 52,174 \quad \sigma = 7500 \quad n = 100$

$$P(\bar{X} < 50000) = P\left(z < \frac{50000 - 52174}{\frac{7500}{\sqrt{100}}}\right)$$

$$P(z < -2.9) = \boxed{.0019}$$

(10) a) $\mu = 29,863 \quad \sigma = 5100 \quad n = 1$

$$P(X > 40000) = P\left(z > \frac{40000 - 29863}{5100}\right) = P(z > 1.99)$$

$$= \boxed{.023}$$

b) $\mu = 29863 \quad \sigma = 5100 \quad n = 80$

$$P(\bar{X} > 30000) = P\left(z > \frac{30000 - 29863}{\frac{5100}{\sqrt{80}}}\right) = P(z > .24)$$

$$= .405$$

(11) a) $\mu = 660 \quad \sigma = 35 \quad n = 1$

$$P(X > 670) = P\left(z > \frac{670 - 660}{35}\right) = P(z > .286) = \boxed{.387}$$

b) $\mu = 660 \quad \sigma = 35 \quad n = 10$

$$P(\bar{X} > 670) = P\left(z > \frac{670 - 660}{35/\sqrt{10}}\right) = P(z > .904) = \boxed{.183}$$

(12) $\mu = 458 \quad \sigma = 97 \quad n = 36$

$$P(450 < x < 465) = P\left(\frac{450-458}{\frac{97}{\sqrt{36}}} < z < \frac{465-458}{\frac{97}{\sqrt{36}}}\right)$$

$$P(-.495 < z < .433) = \boxed{.357}$$

(13) $\mu = 2000 \quad \sigma = 187.5 \quad n = 50$

$$P(1980 < \bar{x} < 1990) = P\left(\frac{1980-2000}{\frac{187.5}{\sqrt{50}}} < z < \frac{1990-2000}{\frac{187.5}{\sqrt{50}}}\right)$$

$$P(-.75 < z < -.38) = \boxed{.125}$$

(14) a) $\mu = 24893 \quad \sigma = 4362 \quad n = 1$

$$P(x < 26000) = P\left(z < \frac{26000-24893}{4362}\right) = P(z < .368)$$

$$= \boxed{.644}$$

b) $\mu = 24893 \quad \sigma = 4362 \quad n = 25$

$$P(\bar{x} < 26000) = P\left(z < \frac{26000-24893}{\frac{4362}{\sqrt{25}}}\right) = P(z < 1.84)$$

$$= \boxed{.967}$$

(15) a) $\mu = 46.2 \quad \sigma = 8 \quad n = 1$

$$P(x < 43) = P\left(z < \frac{43-46.2}{8}\right) = P(z < -.4) = \boxed{.345}$$

b) $\mu = 46.2 \quad \sigma = 8 \quad n = 50$

$$P(\bar{x} < 43) = P\left(z < \frac{43-46.2}{8/\sqrt{50}}\right) = P(z < -2.83) = \boxed{.002}$$