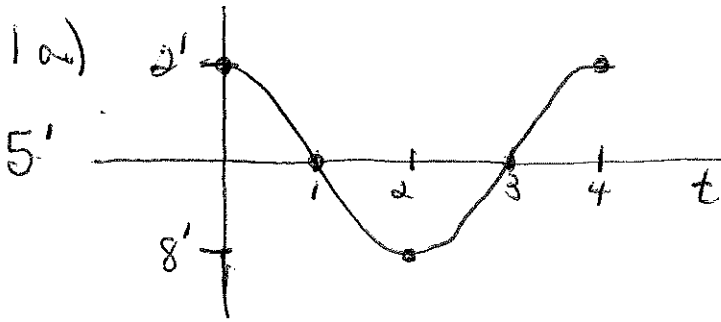


# WS: Harmonic Motion

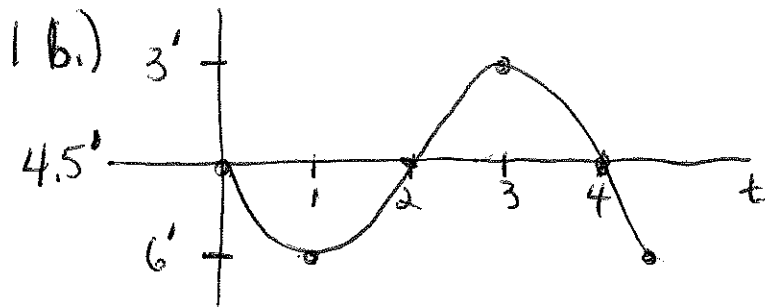


$$h(t) = 3 \cos\left[\frac{\pi}{2}(t-0)\right] + 5$$

$$= 3 \cos\left(\frac{\pi}{2}t\right) + 5$$

$$h(t) = 3 \sin\left[\frac{\pi}{2}(t-3)\right] + 5$$

$$= 3 \sin\left[\frac{\pi}{2}t - 3\frac{\pi}{2}\right] + 5$$



$$h(t) = 1.5 \cos\left[\frac{\pi}{2}(t-3)\right] + 4.5$$

$$= 1.5 \cos\left[\frac{\pi}{2}t - 3\frac{\pi}{2}\right] + 4.5$$

$$h(t) = 1.5 \sin\left[\frac{\pi}{2}(t-2)\right] + 4.5$$

$$= 1.5 \sin\left[\frac{\pi}{2}t - \pi\right] + 4.5$$

Interval on x-axis = 1 sec

$$|A| = \frac{8-2}{2} = 3'$$

$$VS = \frac{8+2}{2} = 5'$$

$$NP = 4 \text{ sec} \quad B = \frac{2\pi}{4} = \frac{\pi}{2}$$

PS = 0 for cosine PS = 3 sec for sine

\* if you use  
PS = 1 for sine, then  
you must have reflect  
across x-axis  
 $h(t) = -3 \sin\left[\frac{\pi}{2}(t-1)\right] + 5$

Interval on x-axis = 1 sec

$$|A| = \frac{6-3}{2} = \frac{3}{2} = 1.5'$$

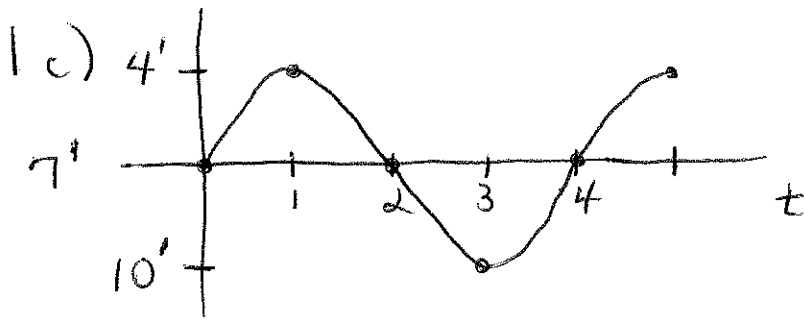
$$VS = \frac{6+3}{2} = \frac{9}{2} = 4.5'$$

$$NP = 4 \text{ sec} \quad B = \frac{2\pi}{4} = \frac{\pi}{2}$$

PS = 3 sec for cosine

PS = 2 sec for sine

or  $h(t) = -1.5 \sin\left[\frac{\pi}{2}(t-0)\right] + 4.5$



Intervals on x-axis = 1 sec

$$VS = 7' \quad 7 = \frac{10 - 4}{2}$$

$$4' = y$$

$$|A| = \frac{10 - 4}{2} = 3'$$

$$h(t) = 3 \cos \left[ \frac{\pi}{2} (t - 1) \right] + 7$$

$$= 3 \cos \left[ \frac{\pi}{2} t - \frac{\pi}{2} \right] + 7$$

$$h(t) = 3 \sin \left[ \frac{\pi}{2} (t - 0) \right] + 7$$

$$= 3 \sin \left[ \frac{\pi}{2} t \right] + 7$$

$$NP = 4 \text{ sec} \quad B = \frac{2\pi}{4} = \frac{\pi}{2}$$

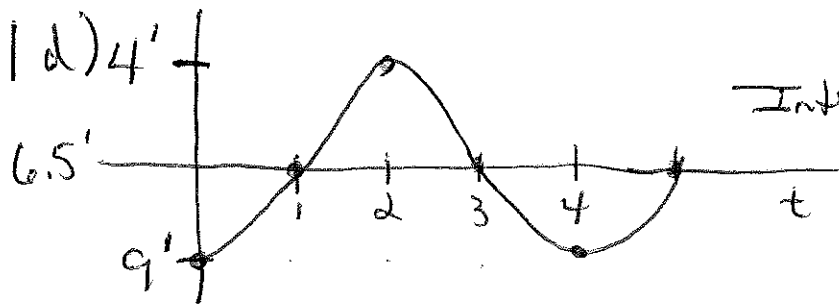
$4 \div 4 = 1 \text{ sec}$  critical pts every 1 sec

PS = 1 sec for cos

PS = 0 for sine

PS = 2 sec for sine

requires reflection



Intervals on x-axis = 1 sec

$$|A| = \frac{9 - 4}{2} = 2.5'$$

$$VS = 9 - 2.5 = 6.5'$$

$$\text{min} = 9 - 5 = 4'$$

$$h(t) = 2.5 \cos \left[ \frac{\pi}{2} (t - 2) \right] + 6.5$$

$$= 2.5 \cos \left[ \frac{\pi}{2} t - \pi \right] + 6.5$$

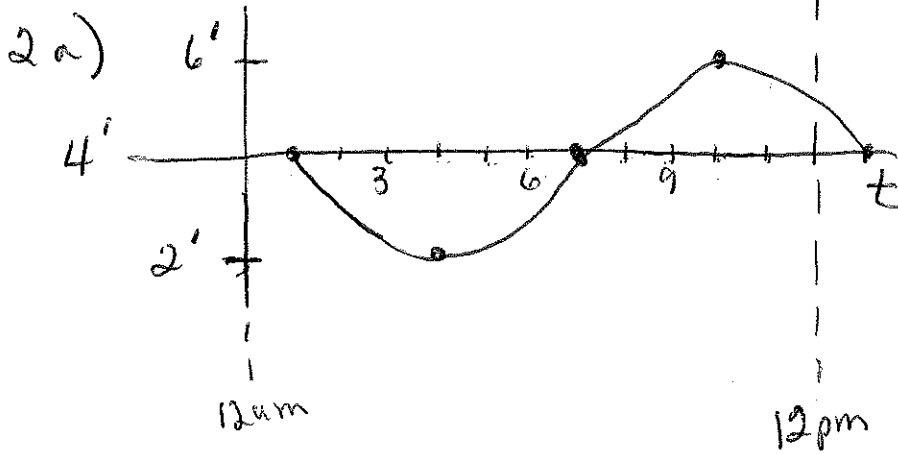
$$NP = 4 \text{ sec} \quad B = \frac{2\pi}{4} = \frac{\pi}{2}$$

$4 \div 4 = 1 \text{ sec}$  critical pts every 1 sec

$$h(t) = 2.5 \sin \left[ \frac{\pi}{2} (t - 1) \right] + 6.5$$

$$= 2.5 \sin \left[ \frac{\pi}{2} t - \frac{\pi}{2} \right] + 6.5$$

PS = 2 sec for cos PS = 1 sec for sine



Interval on x-axis = 1 hr

$$|A| = \frac{6-2}{2} = 2'$$

$$VS = \frac{6+2}{2} = 4'$$

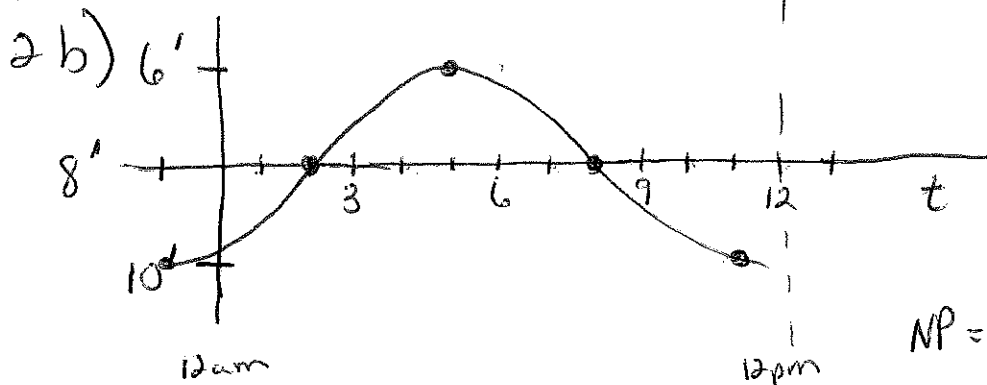
$$NP = 12 \text{ hr} \quad \beta = \frac{2\pi}{12} = \frac{\pi}{6}$$

$12 \div 4 = 3 \text{ hr}$  critical pts every 3 hr

$$\begin{aligned} h(t) &= 2 \cos \left[ \frac{\pi}{6} (t-10) \right] + 4 \\ &= 2 \cos \left[ \frac{\pi}{6} t - \frac{5\pi}{3} \right] + 4 \end{aligned}$$

PS = 10 hrs for cos PS = 7 hr for sin

$$\begin{aligned} h(t) &= 2 \sin \left[ \frac{\pi}{6} (t-7) \right] + 4 \\ &= 2 \sin \left[ \frac{\pi}{6} t - \frac{7\pi}{6} \right] + 4 \end{aligned}$$



Interval on x-axis = 1 hr

$$VS = 8' \quad 8 = \frac{10+6}{2}$$

$$C' = 4$$

$$|A| = \frac{10-6}{2} = 2'$$

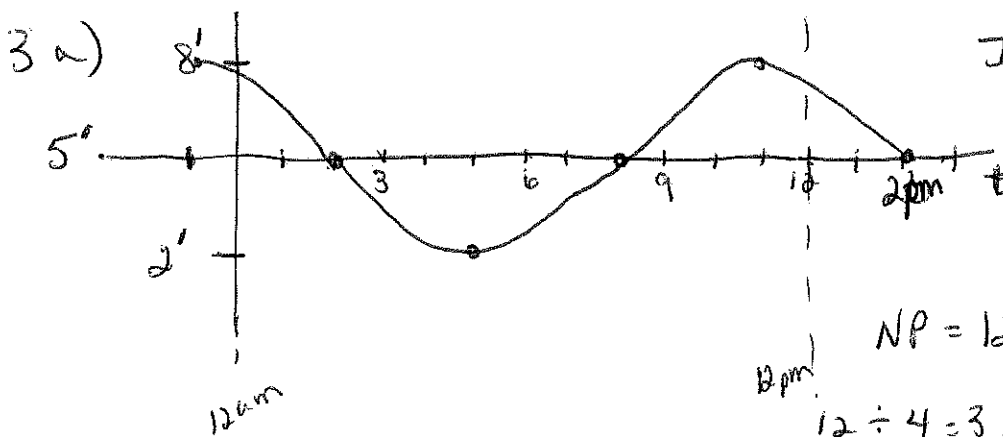
$$NP = 12 \text{ hr} \quad \beta = \frac{2\pi}{12} = \frac{\pi}{6}$$

$12 \div 4 = 3 \text{ hr}$  critical pts every 3 hr

PS = 5 hr for cos PS = 2 hr for sin

$$\begin{aligned} h(t) &= 2 \cos \left[ \frac{\pi}{6} (t-5) \right] + 8 \\ &= 2 \cos \left[ \frac{\pi}{6} t - \frac{5\pi}{6} \right] + 8 \end{aligned}$$

$$\begin{aligned} h(t) &= 2 \sin \left[ \frac{\pi}{6} (t-2) \right] + 8 \\ &= 2 \sin \left[ \frac{\pi}{6} t - \frac{\pi}{3} \right] + 8 \end{aligned}$$



Intervals on x-axis = 1 hr

$$|A| = \frac{8-2}{2} = 3'$$

$$VS = \frac{8+2}{2} = 5'$$

$$NP = 12 \text{ hr} \quad B = \frac{2\pi}{12} = \frac{\pi}{6}$$

12pm

$12 \div 4 = 3 \text{ hr}$  critical pts every 3 hr

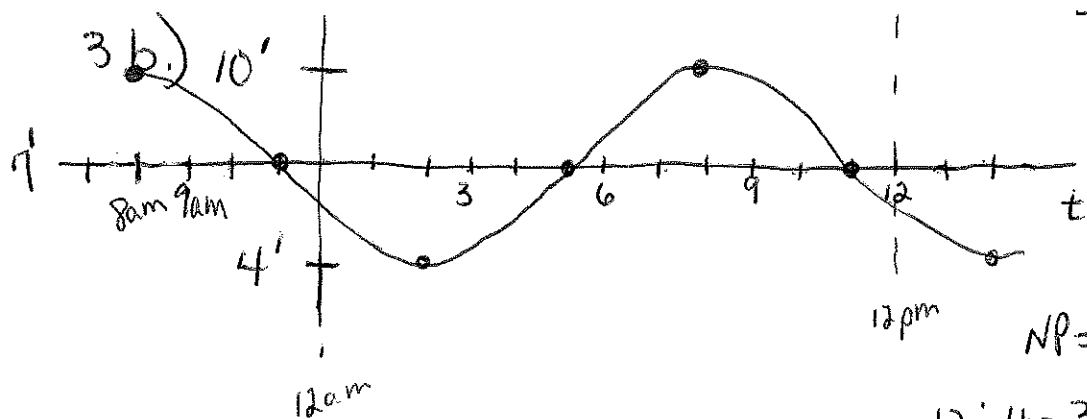
PS = 11 hr for cos PS = 8 hr for sin

$$h(t) = 3 \cos \left[ \frac{\pi}{6} (t - 11) \right] + 5$$

$$= 3 \cos \left[ \frac{\pi}{6} t - \frac{11\pi}{6} \right] + 5$$

$$h(t) = 3 \sin \left[ \frac{\pi}{6} (t - 8) \right] + 5$$

$$= 3 \sin \left[ \frac{\pi}{6} t - \frac{4\pi}{3} \right] + 5$$



Intervals on x-axis = 1 hr

$$VS = 7' \quad 7 = \frac{10+4}{2}$$

$$4' = y$$

$$|A| = \frac{10-4}{2} = 3'$$

$$NP = 12 \text{ hr} \quad B = \frac{2\pi}{12} = \frac{\pi}{6}$$

$12 \div 4 = 3 \text{ hr}$  critical pts every 3 hr

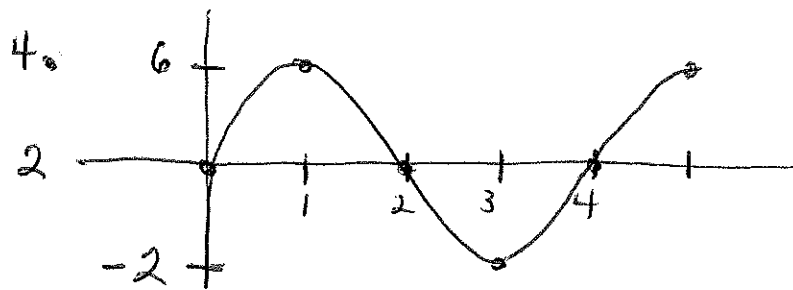
PS = 8 hr for cos PS = 5 hr for sine

$$h(t) = 3 \cos \left[ \frac{\pi}{6} (t - 8) \right] + 7$$

$$= 3 \cos \left[ \frac{\pi}{6} t - \frac{4\pi}{3} \right] + 7$$

$$h(t) = 3 \sin \left[ \frac{\pi}{6} (t - 5) \right] + 7$$

$$= 3 \sin \left[ \frac{\pi}{6} t - \frac{5\pi}{6} \right] + 7$$



Intervals on x-axis = 1

$$|A| = \frac{6 - (-2)}{2} = 4$$

$$VS = \frac{6 + (-2)}{2} = 2$$

$$NP = 4 \quad B = \frac{2\pi}{4} = \frac{\pi}{2}$$

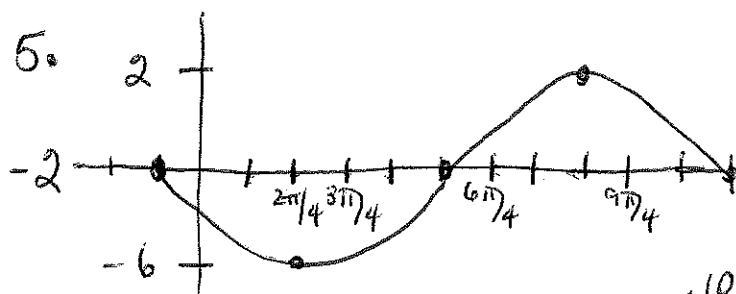
PS = 1 for cos PS = 0 for sine

$$h(t) = 4 \cos\left[\frac{\pi}{2}(t-1)\right] + 2$$

$$= 4 \cos\left[\frac{\pi}{2}t - \frac{\pi}{2}\right] + 2$$

$$h(t) = 4 \sin\left[\frac{\pi}{2}(t-0)\right] + 2$$

$$= 4 \sin\left(\frac{\pi}{2}t\right) + 2$$



Intervals on x-axis =  $\frac{\pi}{4}$

$$|A| = \frac{2 - (-6)}{2} = 4$$

$$VS = \frac{2 + (-6)}{2} = -2$$

$$NP = 3\pi \quad B = \frac{2\pi}{3\pi} = \frac{2}{3}$$

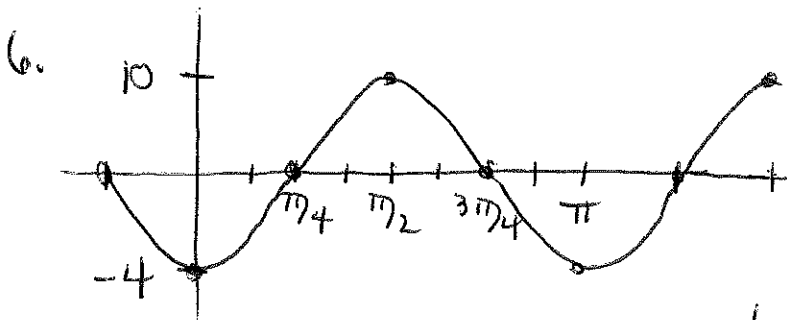
$$h(t) = 4 \cos\left[\frac{2}{3}(t - 2\pi)\right] - 2$$

$$= 4 \cos\left[\frac{2}{3}t - \frac{4\pi}{3}\right] - 2$$

$3\pi \div 4 = \frac{3\pi}{4}$  critical pts every  $\frac{3\pi}{4}$ <sup>50</sup>  
 PS =  $\frac{8\pi}{4} = 2\pi$  for cos PS =  $\frac{5\pi}{4}$  for sin

$$h(t) = 4 \cos\left[\frac{2}{3}(t - \frac{5\pi}{4})\right] - 2$$

$$= 4 \cos\left[\frac{2}{3}t - \frac{5\pi}{2}\right] - 2$$



$$NP = \pi \quad B = \frac{2\pi}{\pi} = 2$$

$\pi \div 4 = \pi/4$  critical pts every  $\pi/4$   
 so intervals on x-axis  
 should be  $\pi/4$ .

$$|A| = \frac{10 - (-4)}{2} = 7 \quad VS = \frac{10 + (-4)}{2} = 3$$

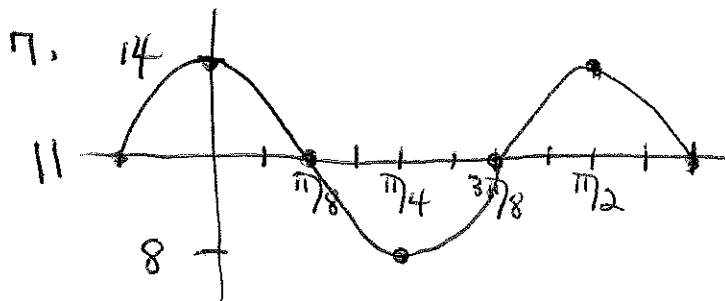
$$PS = \pi/2 \text{ for cos} \quad PS = \pi/4 \text{ for sin}$$

$$h(t) = 7 \cos[2(t - \pi/2)] + 3$$

$$= 7 \cos[2t - \pi] + 3$$

$$h(t) = 7 \sin[2(t - \pi/4)] + 3$$

$$= 7 \sin[2t - \pi/2] + 3$$



$$NP = \pi/2 \quad B = \frac{2\pi}{\pi/2} = 4$$

$\pi/2 \div 4 = \pi/8$  critical pts every  $\pi/8$

$$|A| = \frac{14 - 8}{2} = 3 \quad VS = \frac{14 + 8}{2} = 11$$

$$PS = 0 \text{ for cos} \quad PS = 3\pi/8 \text{ for sin}$$

$$h(t) = 3 \cos[4(t - 0)] + 11$$

$$= 3 \cos[4t] + 11$$

$$h(t) = 3 \sin[4(t - 3\pi/8)] + 11$$

$$= 3 \sin[4t - 3\pi/2] + 11$$