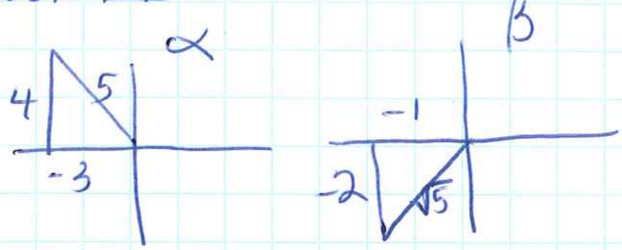
Part II

1) 

a) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$\left(\frac{-3}{5}\right)\left(\frac{-1}{\sqrt5}\right) - \left(\frac{4}{5}\right)\left(\frac{-2}{\sqrt5}\right)$$

$$= \frac{3}{5\sqrt5} + \frac{8}{5\sqrt5} = \frac{11}{5\sqrt5} = \boxed{\frac{11\sqrt5}{25}}$$

* I or IV

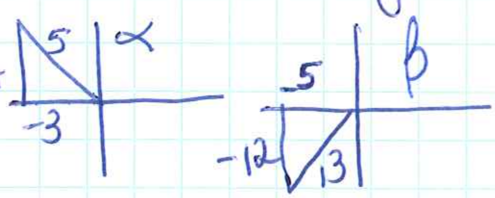
b) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$$\left(\frac{4}{5}\right)\left(\frac{-1}{\sqrt5}\right) + \left(\frac{-2}{\sqrt5}\right)\left(\frac{-3}{5}\right)$$

$$= \frac{-4}{5\sqrt5} + \frac{6}{5\sqrt5} = \frac{2}{5\sqrt5} = \boxed{\frac{2\sqrt5}{25}}$$

* I or II

$\therefore$ The sum of $\alpha+\beta$ exists in quadrant I.

2) 

a) $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

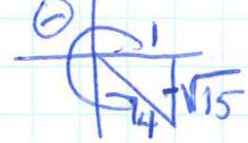
$$= \frac{\left(-\frac{4}{3}\right) + \left(+\frac{12}{5}\right)}{1 - \left(-\frac{4}{3}\right)\left(\frac{-12}{-5}\right)} = \frac{\frac{-20+36}{15}}{1+\frac{48}{15}} = \frac{\frac{+16}{15}}{\frac{15+48}{15}}$$

$$= \frac{\frac{+16}{15}}{\frac{+63}{15}} = \cancel{\frac{16}{15}\cdot\frac{-15}{63} = \frac{16}{11}}$$

$$\frac{16}{15}\cdot\frac{15}{63} = \boxed{\frac{16}{63}}$$

Part II

(3) $\cos\theta = \frac{1}{4}$ $\quad 3\pi/2 < \theta < 2\pi$



a) $\sin\theta = \frac{-\sqrt{15}}{4}$

b) $\sin(\theta - \pi/6) = \sin\theta\cos\pi/6 - \sin\pi/6\cos\theta$

$= \left(\frac{-\sqrt{15}}{4}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)$

$= \frac{-3\sqrt{5}}{8} - \frac{1}{8} = \boxed{\frac{-3\sqrt{5}-1}{8}}$

c) $\cos(\theta + \pi/3) = \cos\theta\cos\pi/3 - \sin\theta\sin\pi/3$

$= \left(\frac{1}{4}\right)\left(\frac{1}{2}\right) - \left(\frac{-\sqrt{15}}{4}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{8} - \left(\frac{-3\sqrt{5}}{8}\right) = \boxed{\frac{1+3\sqrt{5}}{8}}$

d) $\tan(\theta - \pi/4) = \frac{\tan\theta - \tan\pi/4}{1 + \tan\theta\tan\pi/4}$

$= \frac{-\sqrt{15}-1}{1+(-\sqrt{15})(1)} = \frac{-\sqrt{15}-1}{1-\sqrt{15}}$ No Radicals in denominator!

$\frac{-\sqrt{15}-1}{1-\sqrt{15}} \cdot \frac{1+\sqrt{15}}{1+\sqrt{15}} = \frac{-\sqrt{15} - 15 - 1 - \sqrt{15}}{1 + \cancel{\sqrt{15}} - \cancel{\sqrt{15}} - 15}$

$= \frac{-2\sqrt{15} - 16}{-14} = \boxed{\frac{\sqrt{15}+8}{7}}$

Reduce using -2

Part III

1. $\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$

$= (0)(\cos x) + (1)(\sin x)$

$= 0 + \sin x = \boxed{\sin x}$

** Recall: cofunctions!

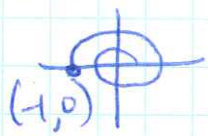
$\cos(90° - \theta) = \sin\theta$

2. $\cos\left(\theta - \frac{3\pi}{2}\right) = \cos\theta\cos\frac{3\pi}{2} + \sin\theta\sin\frac{3\pi}{2}$

$= (\cos\theta)(0) + (\sin\theta)(-1)$

$= 0 - \sin\theta = \boxed{-\sin\theta}$

3. $\tan(\theta + 3\pi) = \dfrac{\tan\theta + \tan 3\pi}{1 - \tan\theta\tan 3\pi}$



$= \dfrac{\tan\theta + 0}{1 - (\tan\theta)(0)} = \dfrac{\tan\theta}{1} = \boxed{\tan\theta}$

4. $\cos\left(\frac{3\pi}{2} - x\right) = \cos\frac{3\pi}{2}\cos x + \sin\frac{3\pi}{2}\sin x$

$= (0)(\cos x) + (-1)(\sin x)$

$= \boxed{-\sin x}$

5. $\cos(\pi + x) = \cos\pi\cos x - \sin\pi\sin x$

$= (-1)(\cos x) - (0)(\sin)x$

$= \boxed{-\cos x}$

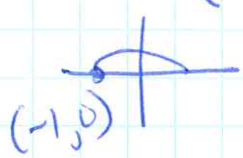
Part III

6. $\sin\left(\frac{3\pi}{2} + \theta\right) = \sin\frac{3\pi}{2}\cos\theta + \cos\frac{3\pi}{2}\sin\theta$

$$= (-1)(\cos\theta) + (0)\sin\theta$$

$$= \boxed{-\cos\theta}$$

7. $\tan(\pi + \theta) = \dfrac{\tan\pi + \tan\theta}{1 - \tan\pi\tan\theta} = \dfrac{0 + \tan\theta}{1-(0)(\tan\theta)}$



$$= \frac{\tan\theta}{1} = \boxed{\tan\theta}$$

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* Verify

1. $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$

$\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x =$

$(0)(\cos x) - (1)\sin x =$

$-\sin x = -\sin x$

2. $\cos 2x = \cos^2 x - \sin^2 x$ [Hint: $\cos 2x = \cos(x+x)$)

$\cos(\overbrace{\underset{\alpha}{x} + \underset{\beta}{x}}) =$

$\cos \cancel{x} \cos x - \sin x \sin x =$

$\cos^2 x - \sin^2 x =$

3. $1 + \cos 2x - \cos^2 x = \cos^2 x$ [Hint: Use previous result.]

$$1 + (\cos^2 x - \sin^2 x) - \cos^2 x =$$

$$1 + \cancel{\cos^2} x - \sin^2 x - \cancel{\cos^2} x =$$

$$1 - \sin^2 x$$

$$\cos^2 x \qquad = \cos^2 x$$