

In Class Examples: Sum and Difference Formulas Part I

$$\textcircled{3} \quad \cos(165^\circ) = \cos(135^\circ + 30^\circ)$$

$$= \cos 135^\circ \cos 30^\circ - \sin 135^\circ \sin 30^\circ$$
$$\left(\frac{-\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$\frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6} - \sqrt{2}}{4}}$$

$$\textcircled{4} \quad \sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$$

$$\sin \frac{\pi}{3} \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos \frac{\pi}{3}$$
$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$$

$$\textcircled{5} \quad \cos(75^\circ) = \cos(45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$
$$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$
$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

⑥ $\sec\left(\frac{\pi}{12}\right)$ * must work as $\cos\left(\frac{\pi}{12}\right)$ then take reciprocal

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) =$$

$$\cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4} = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4} \quad \text{Now FLIP!}$$

* No Radicals in denom

$$\frac{4}{(\sqrt{2} + \sqrt{6})} \cdot \frac{(\sqrt{2} - \sqrt{6})}{(\sqrt{2} - \sqrt{6})} = \frac{4\sqrt{2} - 4\sqrt{6}}{-4} = \boxed{-\sqrt{2} + \sqrt{6}}$$

$$2 - \sqrt{2} - \sqrt{2} - 6$$

⑦ $\csc(15^\circ)$ * must work as $\sin(15^\circ)$ then take reciprocal

$$\sin(15^\circ) = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \sin 30^\circ \cos 45^\circ$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4} \quad \text{Flip!!}$$

No Radicals in denom

$$\frac{4}{\sqrt{6} - \sqrt{2}} \cdot \frac{(\sqrt{6} + \sqrt{2})}{(\sqrt{6} + \sqrt{2})} = \frac{4\sqrt{6} + 4\sqrt{2}}{6 - 2} = \frac{4\sqrt{6} + 4\sqrt{2}}{4} \quad \text{Reduce}$$

$$6 + \sqrt{2} - \sqrt{2} - 2 = \boxed{\sqrt{6} + \sqrt{2}}$$

$$\textcircled{8} \quad \tan(105^\circ) = \tan(60^\circ + 45^\circ)$$

$$= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ} = \frac{\sqrt{3} + 1}{1 - (\sqrt{3})(1)}$$

No radicals
in denom

$$= \frac{(\sqrt{3} + 1)(1 + \sqrt{3})}{(1 - \sqrt{3})(1 + \sqrt{3})} = \frac{\sqrt{3} + 3 + 1 + \sqrt{3}}{1 + \sqrt{3} - \sqrt{3} - 3} = \frac{4 + 2\sqrt{3}}{-2}$$

Reduce

Must
foil both
numerator
& denominator!

$$= \boxed{-2 - \sqrt{3}}$$