Answers Putting It All Together: Probability Models and Sampling Distributions Ch 9

Part I: Discrete Probability Models

A. Binomial Probability Model B(n, p)

Ex 1

- n = 16
- S = has type 0 neg, F = does not have type O neg
- Blood types independent
- Prob remains the same, p = .06, q = .94 B(16, .06)

X = the number of people who have type O neg blood that are randomly selected out of 16 people

a) μ = np = (16)(0.06) = 0.96

On average, in the long run, we can expect 0.96 people out of 16 randomly selected to have type O neg blood.

b) $\sigma = \sqrt{(16)(.06)(.94)} = .9499$

c) P(X = 0) = ${}_{16}C_0(.06)^0(.94)^{16} = .3716$

The probability that no one in a group of 16 randomly selected people have type O neg blood is 37.16%.

d) P(X = 1) + P(X = 2) = ${}_{16}C_1(.06)^1(.94)^{15} + {}_{16}C_2(.06)^2(.94)^{14} = .5611$

The probability that exactly 1 or 2 people in a group of 16 randomly selected people have type O neg blood is 56.11%.

Ex 2

- n = 21
- S = attend minor league game, F = does not attend minor game
- Attending game independent
- Prob remains the same, p = .22, q = .78 B(21, .22)

X = the number of people who have attended a minor league baseball game that are randomly selected out of 21 people

a) μ = np = (21)(0.22) = 4.62 *** NOT large enough for N(μ,σ)

On average, in the long run, we can expect 4.62 people out of 21 randomly selected to have attended a minor league baseball game.

b) $\sigma = \sqrt{(21)(.22)(.78)} = \sqrt{3.604} = 1.8983$

c) $P(X \ge 4) = 1 - P(X \le 3) = 1 - [{}_{21}C_0(.22)^0(.78)^{21} + ... + {}_{21}C_3(.22)^3(.78)^{18}] = .7102$

The probability that at least 4 out of the 21 randomly selected people have been to a minor league baseball game is 71.02%.

d) $P(X \le 2) = {}_{21}C_0(.22)^0(.78)^{21} + \ldots + {}_{21}C_2(.22)^2(.78)^{19} = .1281$

The probability that at most 2 out of 21 randomly selected people have been to a minor league baseball game is 12.81%.

Part I: Discrete Probability Models

B. Approximating the Binomial Distribution using the Normal Model N(μ, σ)

Ex. 1

- n = 170
- S = has type 0 neg, F = does not have type O neg
- Blood types independent
- Prob remains the same, p = .06, q = .94 B(170, .06)

np $\geq$ 10 nq $\geq$ 10
10.2 $\geq$ 10 159.8 $\geq$ 10 Counts large enough to use approx. Normal Distribution

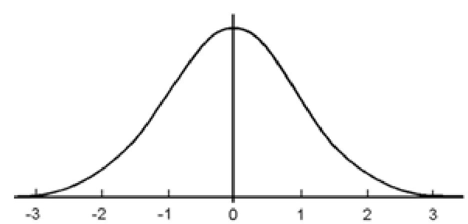
a) μ = np = (170)(0.06) = 10.2

On average, in the long run, we can expect 10.2 people out of 170 randomly selected to have type O neg blood.

b) $\sigma = \sqrt{(170)(.06)(.94)} = 3.10 \quad \rightarrow \quad N(10.2, 3.10)$

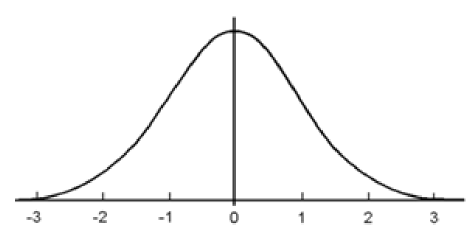
c) $P(X < 9) = P(z < \frac{9 - 10.2}{3.10}) = P(z < -0.3871) = .3493$

The probability that less than 9 out of 170 randomly selected people have type O neg blood is 34.93%.



d) $P(8 < x < 12) = P(\frac{8 - 10.2}{3.1} < z < \frac{12 - 10.2}{3.1}) = P(-0.7097 < z < 0.5806) = 0.4803$

The probability that between 8 and 12 out of 170 randomly selected people have type O neg blood is 48.03%.



Ex. 2

- n = 250
- S = has attended minor league game, F = has not attended minor league game
- attendance independent
- Prob remains the same, p = .22, q = .78 B(250, .22)

np $\geq$ 10 nq $\geq$ 10
55 $\geq$ 10 195 $\geq$ 10 Counts large enough to use approx. Normal Distribution

a) μ = np = (250)(0.22) = 55

On average, in the long run, we can expect 55 people out of 250 randomly selected to have attended a minor league baseball game.

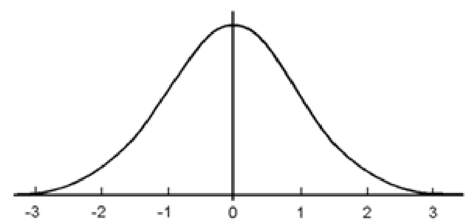
b) $\sigma = \sqrt{(250)(.22)(.78)} = 6.55 \quad \rightarrow \quad N(55, 6.55)$

c) $P(X = 60) = {}_{250}C_{60}(.22)^{60}(.78)^{190} = 0.0443$

The probability that exactly 60 out of 250 randomly selected people have been to a minor league baseball game is 4.43%.

d) $P(X > 70) = P(z > \frac{70-55}{6.55}) = P(z > 2.29) = 0.011$

The probability that more than 70 out of 250 randomly selected people have been to a minor league baseball game is 1.1%.



Part II: Sampling Distributions

A. Mean Sampling Distributions $N(\mu_{\bar{x}}, \sigma_{\bar{x}})$

a) μ = the mean number of days a randomly selected Ford Escort runs before a breakdown

$\mu_{\bar{x}} = \mu = 165\ days \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{64}} = 1.875$

b) State: What is the probability that the number of days before a breakdown for **a randomly selected** Ford Escort is between 164.5 and 164.8 days?

Plan:

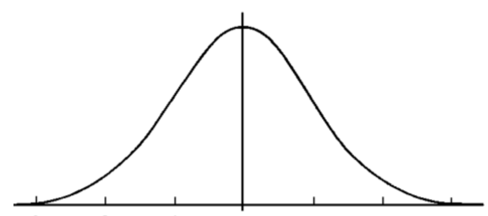
Random: The problem states a random sample of Ford Escorts is taken.

Independence: population of all Ford Escorts $\geq$ 10(64) Condition met for independence (but not needed since selecting a single car).

Large Counts: n = 64 64 $\geq$ 10 CLT states sample size large enough to use approx. Normal Distribution

Do: $P(164.5 < x < 164.8) = P(\frac{164.5-165}{15} < z < \frac{164.8-165}{15}) = P(-0.033 < z < -0.013) = 0.0079$

The probability that a randomly selected Ford Escort goes 164.5 to 164.8 days before a breakdown is 0.79%.



c) State: What is the probability that a randomly selected **sample** of 64 Ford Escorts will have a **mean number of days** before a breakdown less than 160 days?

Plan:

Parameter: μ = the true mean number of days before a breakdown for Ford Escorts

Random: The problem states a random sample of 64 Ford Escorts is taken.

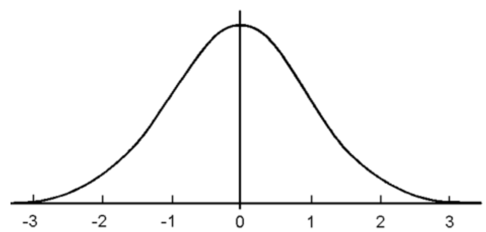
Independence: population of all Ford Escorts $\geq$ 10(64) Condition met for independence.

Large Counts: n = 64 64 $\geq$ 10 CLT states sample size large enough to use approx. Normal Distribution

Do: $\mu_{\bar{x}} = \mu = 165\ days$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{64}} = 1.875$

$$P(\bar{x} < 160) = P(z < \frac{160-165}{1.875}) = P(z < -2.67) = 0.0038$$

The probability that a random sample of 64 Ford Escorts will have a mean number of days before a breakdown less than 160 days is 0.38%.



Ex 2

a) $\mu = 63.7$ $\sigma = 11$ $n = 100$

The mean of the sampling distribution is $\mu_{\bar{x}} = \mu = 63.7.$ The standard deviation of the sampling distribution is

$$\sigma_{\bar{x}} = \frac{11}{\sqrt{100}} = 1.1.$$

b) State: What is the probability that **a randomly selected** student will score less than 61 on the biology exam?

Plan:

Random: The problem states the biology students were randomly selected.

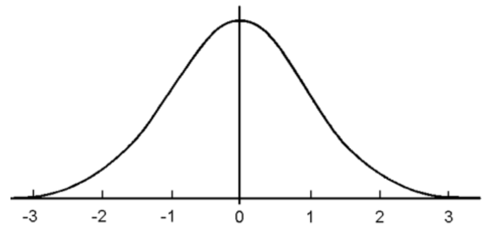
Independence: population of all students taking biology exam $\geq$ 10(100) Condition met for independence (but not needed since selecting a single student).

Large Counts: n = 100 100 $\geq$ 30 CLT says that sample size is large enough to use approx. Normal distribution.

Do: $N(\mu,\sigma)=N(63.7,11)$ $n=1$

$$P(x<61)=P\left(z<\frac{61-63.7}{11}\right)=P(z<-0.245)=0.4032$$

The probability that the grade of a randomly selected student will be less than 61 on a biology exam is 40.32%.



c) State: What is the probability that the **mean grade of a randomly selected group of 100 students** taking a biology exam will be between 65 and 67.2?

Plan:

Parameter: μ = the true mean grade students score on a biology exam

Random: The problem states the biology students were randomly selected.

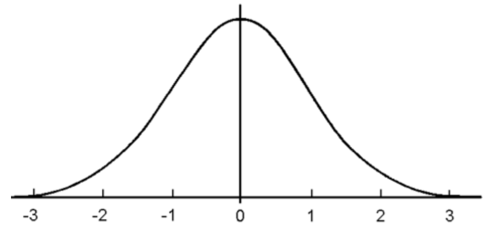
Independence: population of all students taking biology exam $\geq$ 10(100) Condition met for independence.

Large Counts: n = 100 100 $\geq$ 30 CLT says that sample size is large enough to use approx. Normal distribution.

Do: $n=100$ $N\left(63.7,\frac{11}{\sqrt{100}}\right)=N(63.7,1.1)$

$$P(65<\bar{x}<67.2)=P\left(\frac{65-63.7}{1.1}<z<\frac{67.2-63.7}{1.1}\right)=P(1.18<z<3.18)=0.1179$$

The probability that the mean grade on a biology exam of 100 randomly selected students will be between 65 and 67.2 is 11.79%.



Part II: Sampling Distributions

B. Proportion Sampling Distributions $N(\mu_{\hat{p}}, \sigma_{\hat{p}})$

Ex. 1

a) *p* = the true proportion of Maryland residents who believe the Orioles will finish third in their division.

p = .17 n = 130

The mean of the sample proportion is $\mu_{\hat{p}} = .17$.

The standard deviation of the sample proportion is $\sigma_{\hat{p}} = \sqrt{\dfrac{(.17)(.83)}{130}} = .0329.$

b) State: What is the probability that less than 20 of the 130 Maryland residents believe the Orioles will finish third in their division?

Plan:

Parameter: *p* = the true proportion of Maryland residents who believe the Orioles will finish third in their division.

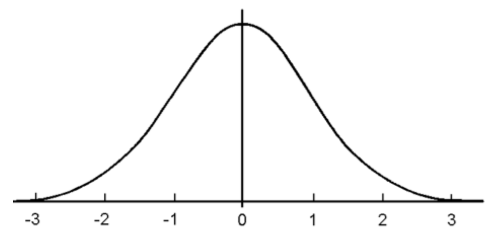
Random: The problem does not state the residents were selected randomly however there is no reason to assume otherwise.

Independence: all residents of Maryland $\geq$ 10(130) Condition met for independence.

Large Counts: np $\geq$ 10 nq $\geq$ 10
22.1 $\geq$ 10 107.9 $\geq$ 10 Sample size large enough to use approx. Normal distribution

Do: $n = 130 \quad \hat{p} = \dfrac{20}{130} = 0.154 \quad \hat{q} = \dfrac{110}{130} = 0.846$

$$P(\hat{p} < .154) = P\left(z < \frac{.154 - .17}{.0329}\right) = P(z < -0.486) = 0.3135$$



The probability that less than 20 of 130 randomly selected residents
In Maryland believe the Orioles will finish third in their division is 31.35%.

c) State: What is the probability that more than 25 of the 130 Maryland residents believe the Orioles will finish third in their division?

Plan:

Parameter: p = the true proportion of Maryland residents who believe the Orioles will finish third in their division.

Random: The problem does not state the residents were selected randomly however there is no reason to assume otherwise.

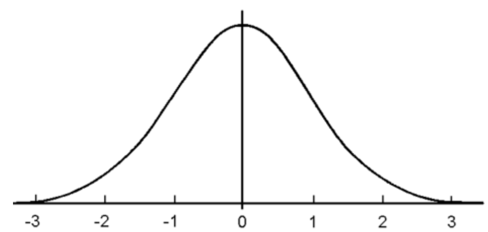
Independence: all residents of Maryland $\geq$ 10(130) Condition met for independence.

Large Counts: np $\geq$ 10 nq $\geq$ 10
22.1 $\geq$ 10 107.9 $\geq$ 10 Sample size large enough to use approx. Normal distribution

Do: $n = 130$ $\hat{p} = \dfrac{25}{130} = 0.1923$ $\hat{q} = \dfrac{105}{130} = 0.8077$

$$P(\hat{p} > .1923) = P\left(z > \frac{.1932 - .17}{.0329}\right) = P(z > 0.678) = 0.2489$$

The probability that more than 25 of 130 randomly selected residents In Maryland believe the Orioles will finish third in their division is 24.89%.



Ex. 2

a) p = the true proportion of Virginia drivers who find traffic conditions unsatisfactory on I-95.

p = .8 n = 400

The mean of the sample proportion is $\mu_{\hat{p}} = .8$.

The standard deviation of the sample proportion is $\sigma_{\hat{p}} = \sqrt{\dfrac{(.8)(.2)}{400}} = .02$.

b) State: What is the probability that between 300 and 315 of 400 Virginia drivers find the traffic conditions on I-95 unsatisfactory ?

Plan:

Parameter: p = the true proportion of Virginia drivers who find the traffic conditions on I-95 unsatisfactory

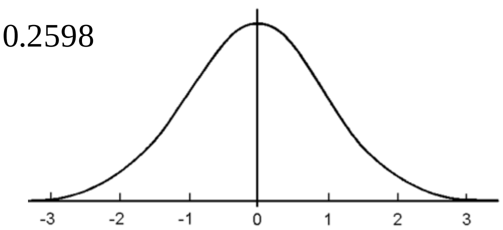
Random: The problem does not state the drivers were selected randomly however there is no reason to assume otherwise.

Independence: all drivers in Virginia $\geq$ 10(400) Condition met for independence.

Large Counts: np $\geq$ 10 nq $\geq$ 10
320 $\geq$ 10 80 $\geq$ 10 Sample size large enough to use approx. Normal distribution

Do: $n = 400$ $\hat{p} = \frac{300}{400} = 0.75$ $\hat{q} = \frac{100}{400} = 0.25$ $\hat{p} = \frac{315}{400} = 0.7875$ $\hat{q} = \frac{85}{400} = 0.2125$

$$P(.75 < \hat{p} < .7875) = P\left(\frac{.75 - .8}{.2} < z < \frac{.7875 - .8}{.2}\right) = P(-2.5 < z < -0.625) = 0.2598$$



The probability that between 300 and 315 of the 400 Virginia drivers find the traffic conditions on I-95 unsatisfactory is 25.98%

c) State: What is the probability that more than 350 of 400 Virginia drivers find the traffic conditions on I-95 unsatisfactory ?

Plan:

Parameter: *p* = the true proportion of Virginia drivers who find the traffic conditions on I-95 unsatisfactory

Random: The problem does not state the drivers were selected randomly however there is no reason to assume otherwise.

Independence: all drivers in Virginia $\geq$ 10(400) Condition met for independence.

Large Counts: np $\geq$ 10 nq $\geq$ 10
320 $\geq$ 10 80 $\geq$ 10 Sample size large enough to use approx. Normal distribution

Do: $n = 400$ $\hat{p} = \frac{350}{400} = 0.875$ $\hat{q} = \frac{50}{400} = 0.125$

$$P(\hat{p} > .875) = P\left(z > \frac{.875 - .8}{.2}\right) = P(z > 3.75) = nearly\ 0$$

The probability that more than 350 of the 400 Virginia drivers find the traffic conditions on I-95 unsatisfactory is nearly 0%.