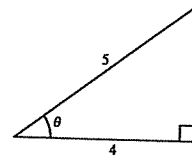


Unit 6: Trig Identities and Equations

Figure for Exercises 1-8

Lesson 4: Double Angle Formulas

Answers HW p. 418 #1 - 8, 21 - 24



$$\begin{aligned}\sin \theta &= \frac{3}{5} \\ \cos \theta &= \frac{4}{5} \\ \tan \theta &= \frac{3}{4}\end{aligned}$$

1. $\sin \theta = \frac{3}{5}$

2. $\tan \theta = \frac{3}{4}$

3. $\begin{aligned}\cos 2\theta &= 2 \cos^2 \theta - 1 \\ &= 2\left(\frac{4}{5}\right)^2 - 1 \\ &= \frac{32}{25} - \frac{25}{25} \\ &= \frac{7}{25}\end{aligned}$

4. $\sin 2\theta = 2 \sin \theta \cos \theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$

5. $\begin{aligned}\tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ &= \frac{2(3/4)}{1 - (3/4)^2} \\ &= \frac{3/2}{1 - (9/16)} \\ &= \frac{3}{2} \cdot \frac{16}{7} \\ &= \frac{24}{7}\end{aligned}$

6. $\begin{aligned}\sec 2\theta &= \frac{1}{\cos 2\theta} \\ &= \frac{1}{\cos^2 \theta - \sin^2 \theta} \\ &= \frac{1}{(4/5)^2 - (3/5)^2} \\ &= \frac{1}{(16/25) - (9/25)} \\ &= \frac{25}{7}\end{aligned}$

7. $\begin{aligned}\csc 2\theta &= \frac{1}{\sin 2\theta} \\ &= \frac{1}{2 \sin \theta \cos \theta} \\ &= \frac{1}{2(3/5)(4/5)} \\ &= \frac{25}{24}\end{aligned}$

8. $\begin{aligned}\cot 2\theta &= \frac{1}{\tan 2\theta} \\ &= \frac{1 - \tan^2 \theta}{2 \tan \theta} \\ &= \frac{1 - (3/4)^2}{2(3/4)} \\ &= \frac{7/16}{3/2} \\ &= \frac{7}{24}\end{aligned}$

21. $\sin u = \frac{3}{5}, 0 < u < \frac{\pi}{2} \Rightarrow \cos u = \frac{4}{5}$

$$\sin 2u = 2 \sin u \cos u = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2(3/4)}{1 - (9/16)} = \frac{24}{7}$$

22. $\cos u = -\frac{2}{7}, \frac{\pi}{2} < u < \pi$ Quadrant II

$$\sin 2u = 2 \sin u \cos u = 2\left(\frac{\sqrt{45}}{7}\right)\left(-\frac{2}{7}\right) = -\frac{12\sqrt{5}}{49}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{4}{49} - \frac{45}{49} = -\frac{41}{49}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2\left(-\frac{\sqrt{45}}{2}\right)}{1 - \frac{45}{4}} = \frac{-\sqrt{45}}{4 - 45} = \frac{12\sqrt{5}}{41}$$

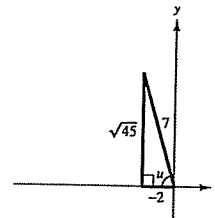
23. $\tan u = \frac{1}{2}, \pi < u < \frac{3\pi}{2} \Rightarrow \sin u = -\frac{1}{\sqrt{5}}$ and

$$\cos u = -\frac{2}{\sqrt{5}}$$

$$\sin 2u = 2 \sin u \cos u = 2\left(-\frac{1}{\sqrt{5}}\right)\left(-\frac{2}{\sqrt{5}}\right) = \frac{4}{5}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \left(-\frac{2}{\sqrt{5}}\right)^2 - \left(-\frac{1}{\sqrt{5}}\right)^2 = \frac{3}{5}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2(1/2)}{1 - (1/4)} = \frac{4}{3}$$



24. $\cot u = -6, \frac{3\pi}{2} < u < 2\pi$ Quadrant IV

$$\sin 2u = 2 \sin u \cos u = 2\left(-\frac{1}{\sqrt{37}}\right)\left(\frac{6}{\sqrt{37}}\right) = -\frac{12}{37}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{36}{37} - \frac{1}{37} = \frac{35}{37}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2\left(-\frac{1}{6}\right)}{1 - \left(-\frac{1}{6}\right)^2} = \frac{-\frac{2}{6}}{\frac{35}{36}} = -\frac{12}{35}$$

