Section $\boxed{9\text{-}3}$

p. 595

(31) $\mu = -3.5\%$ $\sigma = .26\%$ $n = 5$ pop ≥ 10(sample) ✓ all stocks ≥ 50 ✓

a) $\bar{x} \Rightarrow \mu = -3.5\%$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{.26}{\sqrt{5}} = .1163 = 11.63\%$

From all stocks, use σ

b) $P(X \geq 5) = P\left(z \geq \frac{5-(-3.5)}{26}\right) = P(z \geq .327) = .3718 = \boxed{37.18\%}$

From sample size $n=5$ use $\sigma_{\bar{x}}$

c) $P(\bar{x} \geq 5) = P\left(z > \frac{5-(-3.5)}{11.63}\right) = P(z \geq .7309) = \boxed{23.24\%}$

d) lost \$ $\Rightarrow$ below zero

$P(\bar{x} < 0) = P\left(\bar{x} \leq \frac{0-(-3.5)}{11.63}\right) = P(z < .301) = 61.83\%$

Approx 62% of all 5-stock portfolios lost money.

(32) $N(18.6, 5.9)$

a) $P(X \geq 21) = P\left(z \geq \frac{21-18.6}{5.9}\right) = P(z \geq .4068) = .3421$, $\boxed{34.21\%}$

b) $n = 50$ $\mu_{\bar{x}} = \boxed{18.6}$ $\sigma_{\bar{x}} = \frac{5.9}{\sqrt{50}} = \boxed{.8344}$

Results not dependent on individual scores having a normal distribution.

(c) $P(\bar{x} \geq 21) = P\left(Z \geq \frac{21 - 18.6}{.8344}\right) = P(Z \geq 2.876) = .002$

$\boxed{.2\%}$

(33) $\sigma = 10$

a) $\sigma_{\bar{x}} = \frac{10}{\sqrt{3}} = 5.774$ mg

b) Want $\sigma_{\bar{x}} = 3$ $\quad 3 = \frac{10}{\sqrt{n}}$ $\quad n = \frac{100}{9} = 11.\overline{1}$

$9 = \frac{100}{n}$ $\quad \approx \boxed{12 \text{ times}}$

$9n = 100$

(34) $\mu = 6 \quad \sigma = 2.4 \quad n = 10$

a) $\mu_{\bar{x}} \Rightarrow \mu = \boxed{6}$ $\quad \sigma_{\bar{x}} = \frac{2.4}{\sqrt{10}} = \boxed{.7589}$

$\boxed{\mu_{\bar{x}} = 6 \text{ strikes}/km^2}$ $\quad \boxed{\sigma_{\bar{x}} = .7589 \text{ strikes}/km^2}$

b) Not told that it is a normal distribution.
Can't calculate

CLT

p. 601

(35) $\mu = 1.6$ flaws/yd^2 $\sigma = 1.2$ flaws/yd^2

$n = 200$ $\sigma_{\bar{x}} = \frac{1.2}{\sqrt{200}} = .0849$

$P(\cancel{x} > 2) = P\left(z > \frac{2-1.6}{.0849}\right) = P(z > 4.711) = \boxed{0\%}$

(36) $\mu = 13.2\%$ $\sigma = 17.5\%$ $n = 40$ $\sigma_{\bar{x}} = \frac{17.5}{\sqrt{40}} = 2.767$

$P(\bar{x} > 15) = P\left(z > \frac{15-13.2}{2.767}\right) = P(z > .6505) = \boxed{25.77\%}$

$P(\bar{x} < 10) = P\left(z < \frac{15-10}{2.767}\right) = P(z < 1.807) = \boxed{96.46\%}$

(37) $\mu = 190$ lbs $\sigma = 35$ lbs $n = 20$ $\sigma_{\bar{x}} = \frac{35}{\sqrt{20}} = 7.826$ lb

a) NO, we don't know the distribution of the weights

b) $W \Rightarrow$ total weight $\bar{x} = \frac{W}{20}$

CLT says $\bar{x} \approx$ normal $\bar{x} = 190$ $\sigma_{\bar{x}} = \frac{35}{\sqrt{20}} = 7.826$

$P(W > 4000) = P(\bar{x} > 200) = P\left(z > \frac{200-190}{7.826}\right) = P(z > 1.28)$

$= \boxed{10.03\%}$

p602

(38) $\mu = 40.125 \quad \sigma = .002 \quad n = 4 \quad \sigma_{\bar{x}} = \frac{.002}{\sqrt{4}} = .001$

a) $\bar{x} \Rightarrow \mu = 40.125 \quad \sigma_{\bar{x}} = .001$

Normality not needed

b) No, do not know distribution and n = 4 too small for CLT.

(39) $N(125, 10) = X$

a) $P(X > 140) = P\left(Z > \frac{140-125}{10}\right) = P(Z > 1.5) = .0668$ $\boxed{6.68\%}$

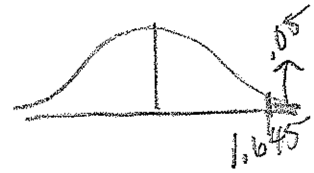
b) $n = 4 \quad \sigma_{\bar{x}} = \frac{10}{\sqrt{4}} = 5 \quad N(125, 5) = \bar{x}$

$P(\bar{x} > 140) = P\left(Z > \frac{140-125}{5}\right) = P(Z > 3) = .0013$ $\boxed{.13\%}$

CLT not needed; Sheila's glucose level follows normal distribution.

(40) $P(\bar{x} > L) = .05 \quad P(Z > z) = .05$

$P(Z > 1.645) = .05$



$1.645 = \frac{L - 125}{5}$

$8.225 = L - 125$

$\boxed{133.225 = L}$

p. 603

(41) $N(298, 3)$ X = ml contained in bottle

a) $P(X < 295) = P\left(z < \frac{295-298}{3}\right) = P(z < -1) = \boxed{15.87\%}$

b) $n = 6$ $\sigma_{\bar{x}} = \frac{3}{\sqrt{6}} = 1.225$

$P(\bar{x} < 295) = P\left(z < \frac{295-298}{1.225}\right) = P(z < -2.45)$
$= \boxed{0.71\%}$

modeled problem (42) $N(55{,}000, 4500)$ X = miles on car for lifetime of brakepads
$n = 8$

a) $\bar{x} \Rightarrow \mu = 55{,}000$ $\sigma_{\bar{x}} = \frac{4500}{\sqrt{8}} = 1590.99$
$N(55{,}000, 1591.0)$

b) $\bar{x} = 51{,}800$ $P(\bar{x} \leq 51{,}800) = P\left(z \leq \frac{51800-55{,}000}{1591}\right)$
$= P(z \leq -2.0113) = .02212 = \boxed{2.2\%}$

(43) $\mu = 2.2$ $\sigma = 1.4$

a) CLT $\Rightarrow N\left(2.2, \frac{1.4}{\sqrt{52}}\right) = N(2.2, .1941)$

b) $P(\bar{x} < 2) = P\left(z < \frac{2-2.2}{.1941}\right) = P(z < -1.0304) = \boxed{15.14\%}$

c) $\bar{x}$ is accidents / week $P\left(\bar{x} < \frac{100}{52}\right) = P(\bar{x} < 1.923)$
$= P\left(z < \frac{1.923-2.2}{.1941}\right) = P(z < -1.471) = .0706 = \boxed{7.1\%}$

p. 604

(44) $\mu = 13.6$ $\quad \sigma = 3.1$ $\quad N(13.6, 3.1)$ $\quad n = 22$

$P(Z < z) = .05$ $\quad P(Z < -1.645)$ $\quad \sigma_{\bar{x}} = \frac{3.1}{\sqrt{22}} \doteq .6609$

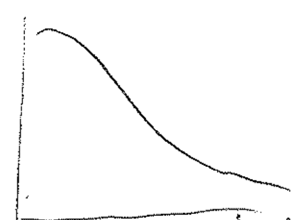
$$-1.645 = \frac{\bar{x} - 13.6}{.6609}$$

$$-1.0872 = \bar{x} - 13.6$$

$$\boxed{12.513 = \bar{x}}$$

(45) Behavior of X less predictable for small number of destinations)
More policies (1,000's) $\bar{x} \Rightarrow \mu$

(46) $\mu = \$250$ $\quad \sigma = \$300$



- Condition met for Normal approximation? CLT sample large enough
- Condition for standard deviation? pop ≥ 10 (sample)
  pop ≥ 10 (

$N\left(250, \frac{300}{\sqrt{10{,}000}}\right)$ $\quad P(\bar{x} \geq 260) = P\left(Z \geq \frac{260-250}{\frac{300}{\sqrt{10{,}000}}}\right) = P(Z \geq 3.33)$
$N(250, 3)$ $\quad = .0004$

p.607

(47) a) parameter $p = 68\%$
statistic $\hat{p} = 73\%$

b) $\mu_{\hat{p}} \Rightarrow p \Rightarrow 68\%$

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.68)(.32)}{150}} = .0381$

c) $P(\hat{p} \geq 73\%) = P\left(z > \frac{.73-.68}{.0381}\right) = P(z \geq 1.331) = .0951$

There appears to be about 10% chance of getting $\hat{p} \geq .73$ if $p = .68$. Random digit device working fine.

(48) Simulation

(49) $N(100, 15)$

a) $P(X \geq 105) = P\left(z \geq \frac{105-100}{15}\right) = P(z \geq 0.3333) = .3695$

b) $\mu_{\bar{x}} = 100 \quad \sigma_{\bar{x}} = \frac{15}{\sqrt{60}} = 1.9365$

c) $P(\bar{x} \geq 105) = P\left(z \geq \frac{105-100}{\frac{15}{\sqrt{60}}}\right) = P(z \geq 2.5819) = .0049$

d) a) would be affected
b) would be the same
c) Reliable $\Rightarrow$ CLT

(50) $p = .47$ $n = 1025$ $\sigma_{\hat{p}} = \sqrt{\frac{(.47)(.53)}{1025}}$

a) Condition for standard deviation met?

pop ≥ 10 (sample)

pop ≥ 10 (1025) ✓

Condition for Normal Approximation met?

$np \geq 10$ $n(1-p) \geq 10$

$(1025)(.47) \geq 10$ ✓ $1025(.53) \geq 10$ ✓

$482 \geq 10$ $543 \geq 10$

$\hat{p}$ approximately normal with $\mu_{\hat{p}} = .47$

$\sigma_{\hat{p}} = .0156$

b) $95\% \Rightarrow \mu_{\hat{p}} \pm 2\sigma_{\hat{p}} = .47 \pm 2(.0156)$

$(.4388, .5012)$

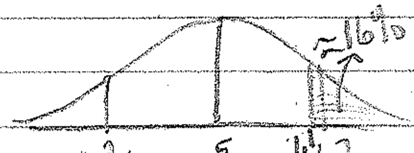
c) $P(\hat{p} < .45) = P\left(z < \frac{.45 - .47}{.0156}\right) = P(z < -1.28)$

$= .1003$

(51) $\mu = 0.5$ $\sigma = 0.7$

a) $\mu_{\bar{x}} = 0.5$ $\sigma_{\bar{x}} = \frac{0.7}{\sqrt{50}} = .09899$

b) CLT



Empirical Rule

$\pm\sigma = 68\%$

Remaining $32\% \div 2$

16%

OR $P(\bar{x} \geq .6) = P\left(z \geq \frac{.6 - .5}{.0989}\right) = P(z \geq 1.011) = 15.6\%$

p. 608

(52) $p = 20.2\%$ $n = 25{,}000$

a) Mean number (COUNT), not proportion.

X = # of H.S. dropouts who receive a flyer.

$\mu_X = np = (.202)(25{,}000) = 5{,}050$

b) $\sigma_X = \sqrt{np(1-p)} = \sqrt{25{,}000(.202)(.798)} = 63.4815$

$P(X \geq 5000) = P\left(z \geq \frac{5000 - 5050}{63.4815}\right) = P(z \geq -.7876) = .7845$

(53) $p = 20\%$ $n = 1555$

a) $np \geq 10$ $n(1-p) \geq 10$

$.2(1555) \geq 10$ $.8(1555) \geq 10$

$311 \geq 10$ $1244 \geq 10$

Normal Approximation

$\mu_X = np = 311$

$\sigma_X = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.2)(.8)}{1555}} = .0101$ pop ≥ 10 (sample)

pop $\geq 15{,}550$

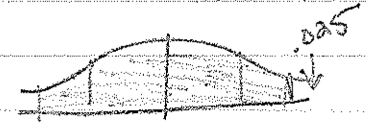
b) $P\left(\hat{p} \leq \frac{300}{1555}\right) = P\left(z < \frac{.1929 - .2}{.0101}\right) = P(z < -.7604) = .2418$

(54) $N(0.2, 0.05)$ $\mu = 0.2$ g/mi $\sigma = 0.05$ g/mi

a) $P(X > 0.3) = P\left(z > \frac{.3 - .2}{.05}\right) = P(z > 2) = .0228$

$= 2.28\%$

or using Empirical Rule 68%-95%-99.7%

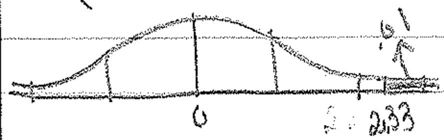


$P(z > 2) = .025 = \boxed{2.5\%}$

b) $n = 25 \quad \mu_{\bar{x}} = 0.2 \text{ g/mi} \quad \sigma_{\bar{x}} = \frac{0.05}{\sqrt{25}} = .01$

$P(\bar{x} > 0.3) = P\left(z > \frac{.3 - .2}{.01}\right) = P(z > 10) = 0\%$

(55) $P(Z > z) = .01 \quad z = 2.33 \quad N(0.2, .01)$



$2.33 = \frac{\bar{x} - .2}{.01}$

$0.2233 = \bar{x}$

$\boxed{0.2233 \text{ g/mi}}$

(56) $\mu = 1.5 \quad \sigma = 0.75$

a) Normal Approximation? no; count must take on whole number values

b) $n = 700$ CLT → Normal Approximation

$\bar{x} \Rightarrow N\left(1.5, \frac{0.75}{\sqrt{700}}\right) \Rightarrow N(1.5, 0.0283)$

c) $P(\bar{x} > 1075) = P\left(\bar{x} > \frac{1075}{700}\right) = P(\bar{x} > 1.5357)$

$= P\left(z > \frac{1.5357 - 1.5}{.0283}\right) = P(z > 1.2615) = .1036 = \boxed{10.36\%}$

(57) $n = 14941 \quad p = .5$

a) $\boxed{\mu_{\hat{p}} \Rightarrow p = .5} \quad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.5)(.5)}{14941}} = \boxed{.0041}$

b) $P(.49 < \hat{p} < .51) = P\left(\frac{.49 - .5}{.0041} < z < \frac{.51 - .5}{.0041}\right)$

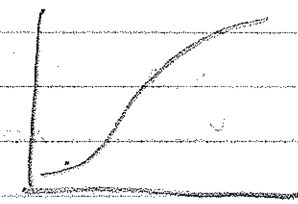
$= P(-2.43 < z < 2.43) = .9849 = \boxed{98.5\%}$

p609

58) $n = 219$ $\bar{x} = 810$

a) $\bar{x}$ is an unbiased estimator of μ if samples of size $n = 219$ were obtained over and over and the sample mean was computed for each distribution, the center of these means would be μ grams)

b) Population distribution normal?
No, more likely skewed left.
Most values will be above a certain value although a few will be below.



$\bar{x}$ birthweight

c) Yes; CLT states that large sample sizes taken from non-normal population distributions results in sampling distributions that are approximately normal.