HW Section 7.2 p.499

7.37 (a) The probability distribution for the new random variable $a+bX$ is shown below.

$a+bX$	5	8	17
$P(a+bX)$	0.2	0.5	0.3

(b) The mean of the new variable is $\mu_{a+bX} = 5\times0.2 + 8\times0.5 + 17\times0.3 = 10.1$, and the variance is $\sigma^2_{a+bX} = (5-10.1)^2\times0.2 + (8-10.1)^2\times0.5 + (17-10.1)^2\times0.3 = 21.69$. (c) The mean of X is $\mu_X =$ 2.7. Using Rule 1 for means, the mean of the new variable is $\mu_{a+bX} = a + b\mu_X = 2 + 3\times2.7 = 10.1$, so the variance will stay the same as the calculation shown in part (b). (d) The variance of X is $\sigma^2_X = 2.41$, so Rule 1 for variances implies that the variance of the new variable is $\sigma^2_{a+bX} = b^2\sigma^2_X = 3^2\times2.41 = 21.69$. This is exactly the same as the variance we obtained in part (b), so $\text{var}(2 + 3X) = \sigma^2_{a+bx} = 9\,\text{var}(X) = 21.69$. (e) Using the rules is much easier than using the definitions. The rules are quicker and enable users to avoiding tedious calculations where mistakes are easy to make.

7.42 (a) The total resistance $T = R_1 + R_2$ is Normal with mean $100 + 250 = 350$ ohms and standard deviation $\sqrt{2.5^2 + 2.8^2} \doteq 3.7537$ ohms. (b) The probability is $P(345 \le T \le 355) =$ $P\left(\frac{345-350}{3.7537} \le Z \le \frac{355-350}{3.7537}\right) = P(-1.332 \le Z \le 1.332) = 0.9086 - 0.0914 = 0.8172$ (Table A gives $0.9082 - 0.0918 = 0.8164$).

7.43 (a) The mean is $\mu_X = 0\times0.03 + 1\times0.16 + 2\times0.30 + 3\times0.23 + 4\times0.17 + 5\times0.11 = 2.68$ toys. The variances of X is $\sigma^2_X = (0 - 2.68)^2\times0.03 + (1 - 2.68)^2\times0.16 + (2 - 2.68)^2\times0.30 + (3 - 2.68)^2\times0.23 + (4 - 2.68)^2\times0.17 + (5 - 2.68)^2\times0.11 \doteq 1.7176$, so the standard deviation is $\sigma_X = \sqrt{1.7176} \doteq 1.3106$ toys. (b) To simulate (say) 500 observations of X, using the T1-83, we will first simulate 500 random integers between 1 and 100 by using the command:

randInt(1,100,500) → L_1

The command sortA(L_1) sorts these random observations in increasing order. We now identify 500 observations of X as follows: integers 1 to 3 correspond to X = 0, integers 4 to 19 correspond to X = 1, integers 20 to 49 correspond to X = 2, integers 50 to 72 correspond to X = 3, integers 73 to 89 correspond to X = 4, and integers 90 to 100 correspond to X = 5. For a sample run of the simulation, we obtained 12 observations of X = 0, 86 observations of X = 1, 155 observations of X = 2, 118 observations of X = 3, 75 observations of X = 4, and 54 observations of X = 5. These data yield a sample mean and standard deviation of $\bar{x} = 2.64$ toys and $s = 1.291$ toys, very close to μ_X and σ_X.

7.46 (a) The mean for the first die (X) is $\mu_X = 1\times1/6 + 3\times1/6 + 4\times1/6 + 5\times1/6 + 6\times1/6 + 8\times1/6 = 4.5$ spots. The mean for the second die (Y) is $\mu_Y = 1\times1/6 + 2\times1/6 + 2\times1/6 + 3\times1/6 + 3\times1/6 + 4\times1/6 = 2.5$ spots. (b) The table below gives the possible values of T = total sum of spots for the two dice. Each of the 36 possible outcomes has probability 1/36.

		Die #1					
		1	3	4	5	6	8
	1	2	4	5	6	7	9
	2	3	5	6	7	8	10
Die	2	3	5	6	7	8	10
#2	3	4	6	7	8	9	11
	3	4	6	7	8	9	11
	4	5	7	8	9	10	12

The probability distribution of T is

t	2	3	4	5	6	7	8	9	10	11	12
P(T=t)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

(c) Using the distribution for (b), the mean is $\mu_T = 2\times1/36 + 3\times2/36 + 4\times3/36 + 5\times4/36 + 6\times5/36 + 7\times6/36 + 8\times5/36 + 9\times4/36 + 10\times3/36 + 11\times2/36 + 12\times1/36 = 7$ spots. Using properties of means (the mean of the sum is the sum of the means) from (a), $\mu_T = \mu_X + \mu_Y = 4.5 + 2.5 = 7$ spots.

7.47 (a) The mean temperature is $\mu_X = 550°\text{C}$. The variance is $\sigma_X^2 = 32.5$, so the standard deviation is $\sigma_X = \sqrt{32.5} \doteq 5.7009\ °\text{C}$. (b) The mean number of degrees off target is 550 – 550 = 0°C, and the standard deviation stays the same, 5.7009°C, because subtracting a constant does not change the variability. (c) In degrees Fahrenheit, the mean is $\mu_Y = \frac{9}{5}\mu_X + 32 = 1022\ °\text{F}$ and the standard deviation is $\sigma_Y = \sqrt{\left(\frac{9}{5}\right)^2 \sigma_x^2} = \left(\frac{9}{5}\right)\sigma_x \doteq 10.2616\ °\text{F}$.

7.48 Read two-digit random numbers from Table B. Establish the correspondence 01 to 10 $\Rightarrow$ 540°, 11 to 35 $\Rightarrow$ 545°, 36 to 65 $\Rightarrow$ 550°, 66 to 90 $\Rightarrow$ 555°, and 91 to 99, 00 $\Rightarrow$ 560°. Repeat many times, and record the corresponding temperatures. Average the temperatures to approximate μ_X; find the standard deviation of the temperatures to approximate σ_X. In one simulation with 200 repetitions, the sample mean of 550.03°C is very close to μ_X and the standard deviation of 5.46°C is slightly smaller than σ_X.

7.49 (a) Yes. The mean of a sum is <u>always</u> equal to the sum of the means. (b) No. The variance of the sum is not equal to the sum of the variances, because it is not reasonable to assume that X and Y are independent.

7.50 (a) The machine that makes the caps and the machine that applies the torque are not the same. (b) Let T denote the torque applied to a randomly selected cap and S denote the cap strength. T is N(7, 0.9) and S is N(10, 1.2), so T – S is Normal with mean 7 – 10 = –3 inch-pounds and standard deviation $\sqrt{0.9^2 + 1.2^2} = 1.5$ inch-pounds. Thus, P(T > S) = P(T – S > 0) = P(Z > 2) = 0.0228.

7.51 (a) The variance of the number of trucks and SUVs is $\sigma_Y^2 = (0 - 0.7)^2 \times 0.4 + (1 - 0.7)^2 \times 0.5 + (2 - 0.7)^2 \times 0.1 = 0.41$ so $\sigma_Y = \sqrt{0.41} \doteq 0.6403$ vehicles. (b) The variance of total sales is $\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 = 0.89 + 0.41 = 1.3$, so the standard deviation of total sales is $\sigma_{X+Y} = \sqrt{1.3} \doteq 1.1402$ vehicles. (c) The variance of Linda's estimated earnings is $\sigma_{350X+400Y}^2 = 350^2 \sigma_X^2 + 400^2 \sigma_Y^2 = 350^2 \times 0.89 + 400^2 \times 0.41 = 174{,}625$, so the standard deviation is $\sigma_{350X+400Y} = \sqrt{174{,}625} \doteq \$417.88$.

7.52 Let L and F denote the respective scores of Leona and Fred. The difference L − F has a Normal distribution with mean $\mu_{L-F} = 24 - 24 = 0$ points and standard deviation $\sigma_{L-F} = \sqrt{2^2 + 2^2} \doteq 2.8284$ points. The probability that the scores differ by more than 5 points is $P(|L-F| > 5) = P\left(|Z| > \frac{5-0}{2.8284}\right) = P(|Z| > 1.7678) \doteq 0.0771$ (Table A gives 0.0768).