HW textbook p.730 #41-49 odd

41. $-x^2 + y^2 + 4x - 6y + 4 = 0 \Rightarrow (y-3)^2 - (x-2)^2 = 1$

$x^2 + y^2 - 4x - 6y + 12 = 0 \Rightarrow (x-2)^2 + (y-3)^2 = 1$

$2y^2 - 12y + 16 = 0$

$2(y-2)(y-4) = 0$

$y = 2$ or $y = 4$

For $y = 2$: $x^2 + 2^2 - 4x - 6(2) + 12 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

For $y = 4$: $x^2 + 4^2 - 4x - 6(4) + 12 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

The points of intersection are (2, 2) and (2, 4).

43. $-4x^2 - y^2 - 16x + 24y - 16 = 0$

$4x^2 + y^2 + 40x - 24y + 208 = 0$

$24x + 192 = 0$

$24x = -192$

$x = -8$

For $x = -8$,

$-4(64) - y^2 - 16(-8) + 24y - 16 = 0$

$-y^2 + 24y - 144 = 0$

$y^2 - 24y + 144 = 0$

$(y-12)^2 = 0 \Rightarrow y = 12$

Solution: $(-8, 12)$

45. $x^2 - y^2 - 12x + 16y - 64 = 0$

$x^2 + y^2 - 12x - 16y + 64 = 0$

$2x^2 - 24x = 0$

$x^2 - 12x = 0$

$x(x-12) = 0 \Rightarrow x = 0, 12$

For $x = 0$, $-y^2 + 16y - 64 = 0$

$y^2 - 16y + 64 = 0$

$(y-8)^2 = 0 \Rightarrow y = 8$

For $x = 12$, $144 - y^2 - 12(12) + 16y - 64 = 0$

$-y^2 + 16y - 64 = 0 \Rightarrow y = 8$

Solutions: (0, 8), (12, 8)

47. $-16x^2 - y^2 + 24y - 80 = 0$

$16x^2 + 25y^2 - 400 = 0$

$24y^2 + 24y - 480 = 0$

$24(y+5)(y-4) = 0$

$y = -5$ or $y = 4$

When $y = -5$: $16x^2 + 25(-5)^2 - 400 = 0$

$16x^2 = -225$

No real solution

When $y = 4$: $16x^2 + 25(4)^2 - 400 = 0$

$16x^2 = 0$

$x = 0$

The point of intersection is (0, 4).

In standard form the equations are:

$$\frac{x^2}{4} + \frac{(y-12)^2}{64} = 1$$

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

49. $x^2 + y^2 = 4$

$3x - y^2 = 0$

Adding $x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0 \Rightarrow x = 1, -4$

For $x = 1$, $y = \pm\sqrt{3}$

$x = -4$ is impossible

Solutions: $(1, \sqrt{3}), (1, -\sqrt{3})$